Solve Equations with a Variable on Both Sides

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Printed: September 15, 2023 (PST)

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Equations with Variables on Both Sides

Move all variables to one side and solve.

Solve Equations with a Variable on Both Sides

[Figure1]

The hockey team is selling chocolate bars for a fundraiser. For the past few weeks, the students have been out there taking orders with the hope of raising enough money for the big tournament. Karen was in charge of sorting through the orders that had come in. She began counting all of the sales that the students had made. She discovered that she and Josie had sold the same amount of boxes. Josie sold thirty-six more boxes than Jake. She sold three times as many as Jake did. So how many boxed did Jake sell?

In this concept, you will learn to solve equations with a **variable** on both sides of the equation.

Equations with a Variable on Both Sides

To solve an equation that has the same variable on both sides of it, you need to get the variables together on one side of the equation, and then get the numbers together on the other side of the equation.

Let's look at an example.

$$12 + t = 30 + 3t$$

Start by subtracting $3t$ and 12 from both sides of the equation.

$$\begin{aligned} 12+t &= 30+3t \\ 12-12+t-3t &= 30-3t-12 \end{aligned}$$

Next, you can combine like terms.

$$\begin{aligned} 12-12+t-3t &= 30-3t-12 \\ -2t &= 18 \end{aligned}$$

Then, solve for the value of the variable by dividing both sides by -2.

$$\begin{aligned} -2t &= 18 \\ \frac{-2t}{-2} &= \frac{18}{-2} \\ t &= -9 \end{aligned}$$

The answer is -9.

Sometimes, an equation will have a set of parentheses and variables on both sides of the equation. The **distributive property** is very helpful in solving these equations.

Let's look at an example.

Solve for 'a':

$$4a+16=13a-(2a+3a)$$

First, simplify the expression on the right side of the equation. According the order of operations, we should combine the like terms inside the parentheses first.

$$\begin{aligned} 4a+16 &= 13a-(2a+3a) \\ 4a+16 &= 13a-(5a) \\ 4a+16 &= 13a-5a \\ 4a+16 &= 8a \end{aligned}$$

Next, notice that the variable, a, is on both sides of the equation. You can use inverse operations to get all of the terms with the variable, a, on one side of the equation. Since there is a number on the left side of the equation and there is no number on the right side of the equation, it is easier to try to get all of the variable terms on the right side of the equation. You can get all of the variable terms on the right side of the equation by subtracting $4a$ from both sides.

$$\begin{aligned} 4a+16 &= 8a \\ 4a-4a+16 &= 8a-4a \\ 16 &= 4a \end{aligned}$$

Then, to solve for 'a', you divide both sides of the equation by 4.

$$\begin{aligned} 16 &= 4a \\ \frac{16}{4} &= \frac{4a}{4} \\ a &= 4 \end{aligned}$$

The answer is 4.

Examples

Example 1

Earlier, you were given a problem about Karen and the chocolate bar orders. She knew the following:

1. She sold as many chocolate bars as Josie.
2. Josie sold thirty-six more boxes than Jake.
3. She sold three times as many as Jake did.

Karen wanted to find out how many chocolate bars Jake sold.

First, you need to write an equation. Let x = the number of chocolate bars that Jake sold. Therefore, Josie sold $(36 + x)$ boxes of bars and Karen sold $3x$ boxes of bars. Therefore the equation is:

$$x + 36 = 3x$$

Next subtract x from both sides of the equation.

$$\begin{aligned} x + 36 &= 3x \\ x - x + 36 &= 3x - x \\ 36 &= 2x \end{aligned}$$

Then, divide both sides by 2 to solve for 'x'.

$$\begin{aligned} 36 &= 2x \\ \frac{36}{2} &= \frac{2x}{2} \\ x &= 18 \end{aligned}$$

The answer is 18.

Jake sold 18 boxes of chocolate bars. This also means that Karen sold 3(18) or 54 boxes of chocolate bars. As well, Josie sold $(18 + 36)$ or 54 boxes of chocolate bars.

Example 2

Solve for 'x' in the equation:

$$6x + 1 = 8x + 3$$

First, you need to move the terms with variables to the same side of the equation. Let's move the $6x$. You can do this by using an **inverse operation**. You subtract $6x$ from both sides of the equation.

$$\begin{aligned} 6x+1 &= 8x+3 \\ 6x-6x+1 &= 8x-6x+3 \\ 1 &= 2x+3 \end{aligned}$$

Next, subtract 3 from both sides of the equation so that only the variable remains on the left hand side.

$$\begin{aligned} 1 &= 2x+3 \\ 1-3 &= 2x+3-3 \\ -2 &= 2x \end{aligned}$$

Then, divide both sides by 2.

$$\begin{aligned} -2 &= 2x \\ \frac{-2}{2} &= \frac{2x}{2} \\ x &= -1 \end{aligned}$$

The answer is -1.

Example 3

$$6x+3=9x+6$$

First, you need to move the terms with variables to the same side of the equation. You can subtract $6x$ from both sides of the equation.

$$\begin{aligned} 6x+3 &= 9x+6 \\ 6x-6x+3 &= 9x-6x+6 \\ 3 &= 3x+6 \end{aligned}$$

Next, subtract 6 from both sides of the equation so that only the variable remains on the left hand side.

$$\begin{aligned} 3 &= 3x + 6 \\ 3 - 6 &= 3x + 6 - 6 \\ -3 &= 3x \end{aligned}$$

Then, divide both sides by 3.

$$\begin{aligned} -3 &= 3x \\ \frac{-3}{3} &= \frac{3x}{3} \\ x &= -1 \end{aligned}$$

The answer is -1.

Example 4

$$4x + x + 2 = 10x - 13$$

First, you should combine like terms on the left side of the equation.

$$\begin{aligned} 4x + x + 2 &= 10x - 13 \\ 5x + 2 &= 10x - 13 \end{aligned}$$

Next, you need to move the terms with variables to the same side of the equation. You subtract $5x$ from both sides of the equation.

$$\begin{aligned} 5x + 2 &= 10x - 13 \\ 5x - 5x + 2 &= 10x - 5x - 13 \\ 2 &= 5x - 13 \end{aligned}$$

Next, add 13 from both sides of the equation so that only the variable remains on the left hand side.

$$\begin{aligned} 2 &= 5x - 13 \\ 2 + 13 &= 5x - 13 + 13 \\ 15 &= 5x \end{aligned}$$

Then, divide both sides by 5.

$$\begin{aligned} 15 &= 5x \\ \frac{15}{5} &= \frac{5x}{3} \\ x &= 3 \end{aligned}$$

The answer is 3.

Example 5

$$8y + 2y = 20y + 10$$

First, you should combine like terms on the left side of the equation.

$$\begin{aligned} 8y + 2y &= 20y + 10 \\ 10y &= 20y + 10 \end{aligned}$$

Next, you need to move the terms with variables to the same side of the equation. You subtract $20y$ from both sides of the equation.

$$\begin{aligned} 10y &= 20y + 10 \\ 10y - 20y &= 20y - 20y + 10 \\ -10y &= 10 \end{aligned}$$

Then, divide both sides by -10.

$$\begin{aligned} -10y &= 10 \\ \frac{-10y}{-10} &= \frac{10}{-10} \\ y &= -1 \end{aligned}$$

The answer is -1.

Review

Solve each equation with variables on both sides.

1. $6x = 2x + 16$
2. $5y = 3y + 12$
3. $4y = y - 18$
4. $8x = 10x + 20$
5. $7x = 4x + 24$
6. $9y = 2y - 21$
7. $-6x + 22 = 5x$
8. $15y = 9y + 36$
9. $14x = 10x - 40$
10. $19y = 4y - 30$
11. $18x = 2x - 32$
12. $4x + 1 = 2x + 5$
13. $6x + 4 = 4x + 10$
14. $8x + 3 = 5x + 9$
15. $10y - 4 = 6y - 12$
16. $8x - 5 = 10x - 13$
17. $12y - 8 = 14y + 14$

18. $18x - 5 = 20x + 19$

19. $-20y + 8 = -8y - 4$

Solve each equation with variables on both sides, by simplifying each equation first by using the distributive property.

20. $2(x + 3) = 8x$

21. $3(x + 5) = -2x$

22. $9y = 4(y - 5)$

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

Vocabulary

Language: English

Term	Definition
distributive property	The distributive property states that the product of an expression and a sum is equal to the sum of the products of the expression and each term in the sum. For example, $a(b + c) = ab + ac$.
Inverse Operation	Inverse operations are operations that "undo" each other. Multiplication is the inverse operation of division. Addition is the inverse operation of subtraction.
Variable	A variable is a symbol used to represent an unknown or changing quantity. The most common variables are a, b, x, y, m, and n.

1.0 REFERENCES

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