Write and Graph Inequalities

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1.7 Write and Graph Inequalities

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[Figure 1]

A dance is being held at the High School and is only open for teenagers who are 18 or younger. Arthur must make posters to advertise the dance in all of the Math classrooms in the school. He wants to make sure the age restriction is included on the poster. How can Arthur mathematically display the age restriction on his posters?

In this concept, you will learn to write and graph inequalities.

Inequalities

An **equation** is a statement of equality between two quantities. A statement that one quantity may or may not be equal to another quantity is called an **inequality.** The statement expressed by an equation is modeled using an equal sign, while the statement expressed by an inequality is modeled using the following symbols:

The symbol $<$ means "is less than."

The symbol $>$ means "is greater than."

The symbol $\leq$ means "is less than or equal to."

The symbol $\geq$ means "is greater than or equal to."

The line below the inequality symbols for less than and greater than indicates the 'equal to' part of the meanings of $\leq$ and $\geq$. If you have difficulty remembering which symbol means less than and which one means greater than, remember "l-e-s-s" points "l-e-f-t," or that the smaller side of the < sign (the point) is the smaller number.

Let's take a closer look at inequalities.

$$2 > x$$

This inequality means "2 is greater than x." There are numerous values for the variable 'x' that would make this statement true. Some of the numbers that could be substituted in for the variable can be listed as shown here: $\{1, 0, -1, \ldots\}$ These values belong to the integers which are a subset of the real numbers.

Another way to show these same numbers is by using set notation. **Set notation** is a mathematical statement that shows an equation or inequality and the set of numbers to which the variable belongs. For the above inequality the values for the variable 'x' can be represented using set notation as shown below:

$$\{x \ : \ 2 > x, \ x \in \mathbb{I}\}$$

$$\left\{x \underbrace{|}_{\text{such that}} 2 > x, x \underbrace{\in}_{\text{belongs to}} \overbrace{\mathbb{I}}^{\text{Integers}}\right\}$$

Read this as: "The set of x values, such that two is greater than x and x is an element of the set of Integer numbers"

Let's look at another example.

$$5 \leq y$$

This inequality means "5 is less than or equal to y." OR "y is greater than or equal to 5." Some of the numbers that could be substituted in for the variable 'y' are $\left\{5, 5.5, 6, 7\frac{1}{4}, \cdots\right\}$ These values belong to the set of rational numbers, which make up a subset of the real numbers. To represent the values for the variable using set notation you could write $\{y \ : \ 5 \leq y, \ y \in \mathbb{Q}\}$. The letter $\mathbb{Q}$ represents the rational numbers. The letter $\mathbb{R}$ could also be used here to represent the real numbers.

Inequalities can also be graphed on a number line. The graph visually displays the set of numbers for the variable that would create a true statement. Remember there are four symbols that can be used to write an inequality. Each of these can be displayed on the number line graph of the inequality. Here are some hints to use when graphing an inequality on a number line:

- If the inequality is written with either a "less than" $(<)$ or a "greater than" $(>)$ symbol, then the starting number of your graph is not included in the solution of the inequality. This is represented by placing an open circle $(\circ)$ on that value on the number line.
- If the inequality is written with either a "less than or equal to" $(\leq)$ or a "greater than or equal to" $(\geq)$ symbol, then the starting number of your graph is included in the solution of the inequality. This is represented by placing a closed circle $(\bullet)$ on that value on the number line.
- To represent "less than" draw an arrow on the number line pointing left. To represent "greater than" draw an arrow on the number line pointing right.
- If the values of the variable belong to the rational numbers or to the real numbers, draw a line joining the plotted values. If the values belong to the natural numbers, whole numbers, or integers do not join the plotted points.

Write an inequality to represent the set of all possible values of 'n' if n is less than two. Then, graph the inequality on a number line.

First, write an inequality to represent the given information.

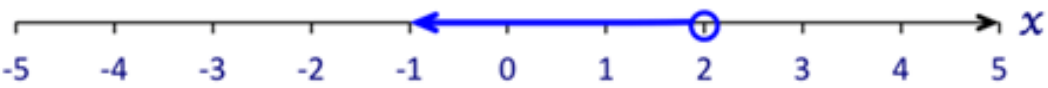
$$\underbrace{n}_{n} \underbrace{\text{is less than}}_{<} \underbrace{2}_{2}$$

The inequality is $n < 2$. This inequality must be graphed on a number line.

First, draw a number line numbered from -5 to 5.

Next, the number 2 is not included in the solution since the values for the variable will be all numbers less than 2. Draw an open circle on the number line at '2'.

Then, draw a direction line to the left from 2 to indicate "less than" and the fact that all numbers less than 2 are included in the solution. This means that $n \in \mathbb{R}$.



[Figure 2]

Examples

Example 1

Earlier, you were given a problem about Arthur and his dance posters. He wants to make sure that all students know the age requirements for attending the dance. How can Arthur do this mathematically?

First, create an eye-catching poster advertising the dance.

Next, write a verbal model to represent the age restriction. "You must be between the ages of 13 and 18."

Verbal Model: To be admitted to the dance your age must be “greater than or equal to 13” and “less than or equal to 18.”

Next, name the variable. Let ‘ a ’ represent the age of the students.

Next, use set notation to write an inequality to represent the verbal model.

$$\{a \ : \ 13 \leq a \leq 18, \ a \in \mathbb{R}\}$$

Example 2

Write an inequality to represent the set of all possible values of ‘ n ’ if n is greater than or equal to negative four. Then, graph the inequality on a number line.

First, write an inequality to represent the given information.

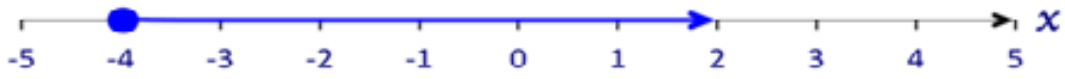
$$\underbrace{n}_{n} \ \underbrace{\text{is greater than or equal to}}_{\geq} \ \underbrace{\text{negative four}}_{-4}$$

The inequality is $n \geq -4$. This inequality must be graphed on a number line.

First, draw a number line numbered from -5 to 5.

Next, the number -4 is included in the solution since the values for the variable will be all numbers greater than or equal to -4. Draw a closed circle on the number line at '-4'.

Then, draw a direction line to the right from -4 to indicate "greater than" and the fact that all numbers greater than -4 are included in the solution. This means that $n \in \mathbb{R}$.



[Figure 3]

Example 3

Draw a number line graph to represent the following inequality expressed in set notation.

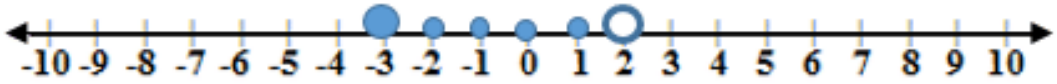
$$\{x \ : \ 3 \leq x < 2, \ x \in \mathbb{I}\}$$

First, write in words what the set notation represents. Remember to read it from the middle beginning with the variable.

The set notation represents all integers "greater than or equal to -3" and "less than" 2.

Next, draw a number line and place a closed circle on -3 since it is included in the solution and an open circle on 2 since it is not included in the solution.

Then, since the integers between these values are included in the solution place closed circles on {-2, -1, 0, and 1}. Do not join the circles with a line since 'x' belongs to the Integers.



[Figure 4]

Example 4

Write an inequality to represent the following statement:

The quantities less than or equal to four.

First, name the variable. Let 'x' represent the quantities in the set.

Next, write the symbol that means "less than or equal to." $\leq$

Next, write the number. 4

Then, write the inequality.

$$x \leq 4$$

The answer is $x \leq 4$.

Example 5

Represent the following using set notation:

All real numbers greater than -2 and less than or equal to 11.

First, name the variable. Let 'n' represent all real numbers in the set.

Next, begin the format for writing set notation. $\{n\ :\ \}$

Next, write the smaller number given in the statement.

$$\{n\ :\ \text{-}2\}$$

Next, insert the symbol that means "less than."

$$\{n\ :\ \text{-}2 <\}$$

Next, write the variable.

$$\{n\ :\ \text{-}2 < n\}$$

Next, write the symbol that means "less than or equal to."

$$\{n\ :\ \text{-}2 < n \leq\}$$

Next, write the larger number given in the statement.

$$\{n\ :\ \text{-}2 < n \leq 11,\}$$

Then, write the set of numbers to which the variable belongs.

$$\{n : -2 < n \leq 11, n \in \mathbb{R}\}$$

The answer is $\{n : -2 < n \leq 11, n \in \mathbb{R}\}$.

Review

Write a solution set for each inequality. Include at least three values in your solution set.

1. $x < 13$
2. $y > 5$
3. $x < 2$
4. $y > -3$
5. $a > 12$
6. $x \leq 4$
7. $y \geq 3$
8. $b \geq -3$
9. $a \leq -5$
10. $b \geq 11$

Write an inequality to describe each situation.

11. A number is less than or equal to -8.
12. A number is greater than 50.
13. A number is less than -4.
14. A number is greater than -12.
15. A number is greater than or equal to 11.

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

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