Recognizing Permutations

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10.4 Recognizing Permutations

FlexBooks 2.0 > VUB Math > Recognizing Permutations

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[Figure 1]

Sandra, Janet, and Karen want to have their picture taken. The photographer wants to put them in a line for the picture, but the girls can't decide who should be in the middle and how they should arrange themselves. They ask the photographer if they can take a shot in every possible arrangement and then choose their favorite. How many different arrangements can the three girls be in for their picture?

In this concept, you will learn to recognize permutations in situations where order is important.

Permutations

Order is important in some situations and not important in others. For example, in following a cake recipe, the order in which the events take place is important. You need to crack the eggs before you mix them with the flour. Similarly, you put the icing on the cake only after it has baked.

In buying the ingredients to make a cake, on the other hand, order is not important. Does it matter if you buy the flour before the eggs or the milk before the icing? It doesn't, so you would say that order is not important in buying cake ingredients.

For solving many problems in which order is important, you can use **permutations**. A permutation is a selection of items in which order is important. To use permutations to solve problems, you need to be able to identify the problems in which order, or the arrangement of items, matters.

Let's look at an example.

Jack has a combination lock on his locker. When opening his locker, does order matter? Is this a permutation?

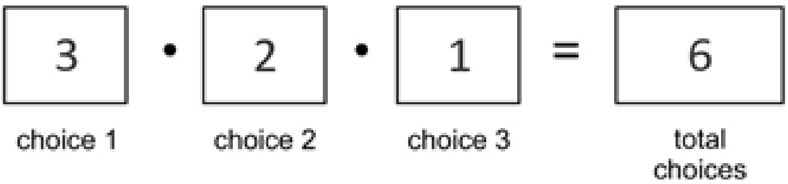
The lock combination is particular to each lock. For example if Jack's combination was $35-17-23$, it would only open if he uses these numbers in this order. Order is therefore important and this is a permutation.

You can count permutations too. There is a mathematical way to calculate the number of permutations possible given the number of items selected.

To count the number of permutations in a problem you need to look at the problem as a series of choices. You can find the number of permutations in a group if you include all members of that group.

Let's look at an example.

Suppose there are 3 cabs in front of a hotel, Acme, Bluebird, and Checker. If all 3 line up to wait for the next customer, the number of different lineups, or permutations, of 3 items taken 3 at a time is:

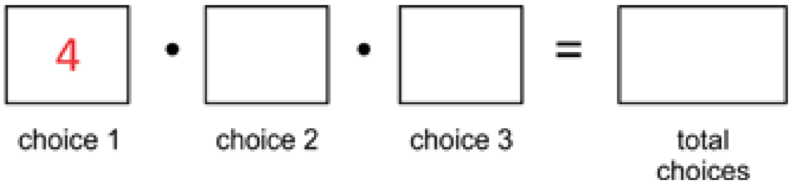


[Figure 2]

Again, this is the permutation for three cabs lined up three at a time. You could also say that this is three objects taken three at a time.

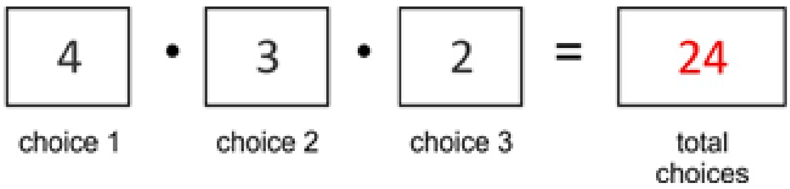
What happens when 4 cabs show up at the hotel, but there is only room for 3 cabs to line up? For example, how many different 3-cab lineups would there be if you started with 4 cabs – Acme, Bluebird, Checker, and Decker?

Now, for choice 1 you have four choices instead of three.



[Figure 3]

For the choice 2 you have three 3 choices instead of two and for choice 3 you have two choices instead of one. The final calculation gives 24 total choices:



[Figure 4]

This is the answer when you have four options taken three at a time.

Examples

Example 1

Earlier, you were given a problem about Janet, Sandra and Karen having their picture taken.

Three girls are getting their picture taken and the photographer wants them to line up.

You need to determine the number of permutations there are for the three girls to be in a line.

First, look at the problem.

If one of the three girls is put in the first spot, two of the girls will be placed in the second and the last girl remaining will be placed in the third spot.

Next, calculate the number of choices.

$$\begin{aligned} P(\text{spot } 1, \text{spot } 2, \text{spot } 3) &= 3 \times 2 \times 1 \\ P(\text{spot } 1, \text{spot } 2, \text{spot } 3) &= 6 \end{aligned}$$

The answer is 6.

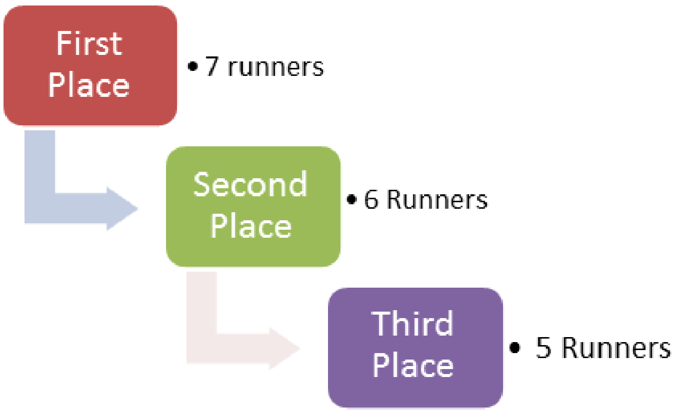
There are 6 different ways Sandra, Karen, and Janet could be lined up for their picture.

Example 2

Seven people are in a race. Determine how many choices there are for first, second, and third place.

First, let's think about the problem. For first place, any seven of the runners could possibly win. For second place, six of the remaining runners could come in second. Five of the remaining runners could come in third.

Next, find the number of choices for first, second, and third.



[Figure 5]

Then, calculate the number of choices.

$$
\begin{aligned}
P(\text{first, second, third}) &= 7 \times 6 \times 5 \\
P(\text{first, second, third}) &= 210
\end{aligned}
$$

The answer is 210.

There are 210 different ways these seven runners could take first, second, and third place in the race.

Example 3

If five people want to go to the movies, but there are only two seats available, how many ways can the people sit if they can only sit two at a time?

First, let's think about the problem.

There are only two seats and there are five people. Any of the five people can be the first person to sit in one of the seats. Any of the remaining four people could take the second of the two seats.

Next, calculate the number of choices.

$$\begin{aligned} P(\text{seat 1, seat 2}) &= 5 \times 4 \\ P(\text{seat 1, seat 2}) &= 20 \end{aligned}$$

The answer is 20.

There are 20 different ways these five people could sit in the two available seats at the movies.

Example 4

How many different ways could the people be seated if there were three seats?

First, let’s think about the problem.

There are only three seats and there are five people. Any of the five people can be the first person to sit in one of the seats. Any of the remaining four people could take the second of the three seats. The remaining three people could be seated in the final empty seat.

Next, calculate the number of choices.

$$\begin{aligned} P(\text{seat 1, seat 2, seat 3}) &= 5 \times 4 \times 3 \\ P(\text{seat 1, seat 2, seat 3}) &= 60 \end{aligned}$$

The answer is 60.

There are 60 different ways these five people could sit in the three available seats at the movies.

Example 5

How many different ways could the people be seated if there were four seats?

First, let’s think about the problem.

There are only four seats and there are five people. Any of the five people can be the first person to sit in one of the seats. Any of the remaining four people could take the second of the four seats. Three people remain for the last two seats. The remaining two people could be seated in the final empty seat.

Next, calculate the number of choices.

$$\begin{aligned} P(\text{seat 1, seat 2, seat 3, seat 4}) &= 5 \times 4 \times 3 \times 2 \\ P(\text{seat 1, seat 2, seat 3, seat 4}) &= 120 \end{aligned}$$

The answer is 120.

There are 120 different ways these five people could sit in the four available seats at the movies.

Review

Figure out each permutation or each outcome.

1. Four different frogs entered the jumping contest -Spots, Dots, Slimey, and Croaky. In how many ways can the 4 finish in first, second, third, and fourth place?

2. Denise has the letters A, R, X, O, G, I, and L. How many different 4-letter arrangements can she make?

3. Six people have signed up to play Scrabble –Tim, Jim, Kim, Pam, Sam, and Cam. Only 4 people can play at one time. How many different 4-player games are possible?

4. Arnold printed out his 8-page report without putting page numbers on the pages. Now the pages have gotten all mixed up. In how many different ways can Arnold arrange the 8 pages?

5. The special lunch at Bamboo Restaurant gives you a choice of won-ton or hot-sour soup, a choice of kung pao shrimp or chicken with broccoli, and a choice rice or noodles. How many different special lunches are there?

6. Rex forgot the password for his ATM bank card. He knows that the password is made of the 4 digits of his birthday, October 24 or 1, 0, 2, 4.? How many different passwords does he need to try to be sure he gets his password?

7. Javier wrote out wrote out the letters of Jasmine's name on 7 cupcakes and put them in a box. How many different ways can he take them out of the box one by one?

8. On the new reality show, Lazybones, the five finalists –Snoozin' Betty, Lounge Man, The Yawn Meister, Lana Later, and Bob the Procrastinator –compete to see who is the laziest person. On today's show, the five will be weeded down to 3 super-finalists. How many different ways can the 3 super-finalists be chosen?

9. Four friends have printed the letters M, E, T, S on the front of their shirts. They're going to the Mets game and will sit in 4 seats side by side. In how many different ways can the four sit?

10. How many different 3-digit numbers can you form from the digits 4, 5, and 6?

Use permutations to solve each problem.

11. How many 3-digit numbers can Blanche write using the digits 7, 8, and 9?

12. How many 4-letter arrangements can Blanche write using the letters A, B, C, D?

13. The programmer for Oddball-TV has 5 new half-hour shows she wants to air on Tuesday evenings: Strange Days, Slightly Off, Odd Rod, Bent, and Icky the Great. In how many different orders can she present the shows?

14. A train has 6 different cars –a passenger car, a baggage car, a mail car, a diner car, a freight car, and a caboose. In how many different orders can the cars be arranged?

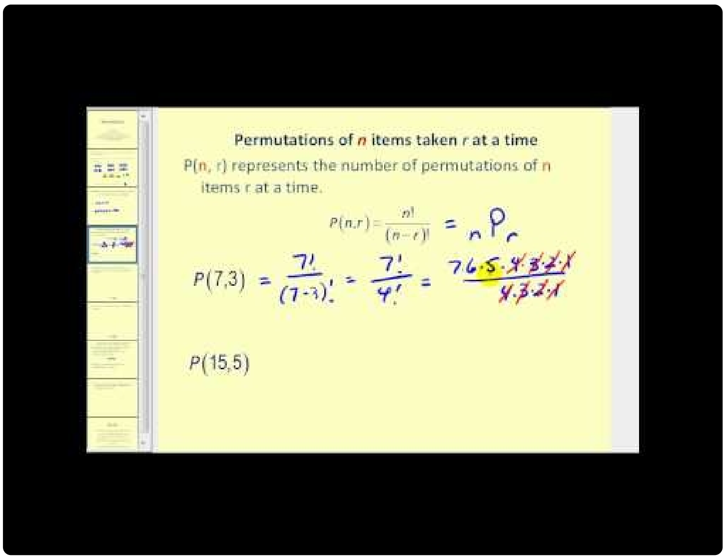
15. The last 5 digits of Beryl's phone number are 34567. How many numbers have these same 5 digits?

16. Mia has 7 charms for her charm bracelet –a heart, a moon, a turtle, a cube, a bird, a hoop, and a car. Into how many different orders can she arrange the 7 charms?

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

Resources



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