Solve Multi-Step Equations Involving Fractions

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Multi-Step Equations with Fractions

Solve equations including fractional values

Solve Multi-Step Equations Involving Fractions

[Figure1]

Kathleen is getting in shape for the “Relay for Life” walk. She tells her brother, Mack, that on Monday she walked 4 miles and on Tuesday one-third as many miles as she walked on Wednesday, for a total of 24 miles. How can Mack create an equation to figure out how far his sister walked on each day?

In this concept, you will learn to solve multi-step equations involving fractions.

Multi Step Equations with Fractions

Before you begin to solve equations involving fractions, you may have to review the rules for operations with fractions. Here are a few tips on how to perform operations with fractions:

- To add or subtract fractions they must have the same **denominator**.

- To multiply fractions you multiply the numerators and write the product over the product of the denominators.
- To divide fractions you multiply the first **fraction** by the reciprocal of the fraction after the division sign.

Let's look at a problem with fractions before you begin solving equations involving fractions.

$$\left(\frac{1}{4}+\frac{1}{2}-\frac{2}{3}\right)\left(\frac{3}{4}\div\frac{1}{2}\right)$$

First, perform the addition and subtraction in the first set of parenthesis. The **least common denominator** is 12.

$$\left[\frac{1}{4}\left(\frac{3}{3}\right)+\frac{1}{2}\left(\frac{6}{6}\right)-\frac{2}{3}\left(\frac{4}{4}\right)\right]$$

Next, multiply each fraction by multiplying the numerators and the denominators.

$$\left(\frac{3}{12}+\frac{6}{12}-\frac{8}{12}\right)$$

Next, express the fractions as a single fraction.

$$\left(\frac{3+6-8}{12}\right)$$

Then, do the addition and subtraction in the numerator.

$$\left(\frac{1}{12}\right)$$

This answer is $\frac{1}{12}$.

Next, perform the division in the second set of parenthesis.

$$\left(\frac{3}{4} \div \frac{1}{2}\right)$$

First, express the division as the first fraction multiplied by the reciprocal of the second fraction.

$$\left(\frac{3}{4} \times \frac{2}{1}\right)$$

Then, multiply the numerators and multiply the denominators.

$$\left(\frac{6}{4}\right)$$

This answer is $\left(\frac{6}{4}\right)$.

Next, multiply the answers from each set of parenthesis.

$$\left(\frac{1}{12}\right)\left(\frac{6}{4}\right)$$

Remember, multiply the numerators and multiply the denominators.

$$\frac{6}{48}$$

Then, simplify the answer.

$$\frac{6}{48} = \frac{1}{8}$$

The answer is $\frac{1}{8}$.

Let's look at solving an equation involving fractions.

Solve the following equation for 'x'.

$$x - \frac{x}{2} - \frac{1}{12} = \frac{5}{6}$$

First, isolate the variable 'x' by adding $\frac{1}{12}$ to both sides of the equation.

$$x - \frac{x}{2} - \frac{1}{12} + \frac{1}{12} = \frac{5}{6} + \frac{1}{12}$$

Next, simplify both sides of the equation.

$$\begin{aligned} x - \tfrac{x}{2} &= \tfrac{10}{12} + \tfrac{1}{12} \\ x - \tfrac{x}{2} &= \tfrac{11}{12} \end{aligned}$$

Next, simplify the left side of the equation by performing the subtraction.

$$\begin{aligned} \tfrac{x}{1}\left(\tfrac{2}{2}\right) - \tfrac{x}{2} &= \tfrac{11}{12} \\ \tfrac{2x}{2} - \tfrac{x}{2} &= \tfrac{11}{12} \\ \tfrac{1x}{2} &= \tfrac{11}{12} \end{aligned}$$

Next, multiply both sides of the equation by 12.

$$\begin{aligned} \left(\tfrac{6}{\cancel{12}}\right)\left(\tfrac{1x}{\cancel{2}}\right) &= \left(\tfrac{1}{\cancel{12}}\right)\left(\tfrac{11}{\cancel{12}}\right) \\ 6x &= 11 \end{aligned}$$

Then, divide both sides of the equation by 6.

$$\begin{aligned} \frac{\overset{1}{\cancel{6}}x}{\cancel{6}} &= \frac{11}{6} \\ x &= 1\frac{5}{6} \end{aligned}$$

The answer is $x = 1\frac{5}{6}$.

Some equations involving fractions will require you to apply the distributive property to clear parenthesis as you solve the equation.

Solve the following equation:

$$\frac{2}{3}\left(y + \frac{3}{5}\right) = 2$$

First, clear the parenthesis by applying the distributive property.

$$\begin{aligned} \tfrac{2}{3}\left(\tfrac{y}{1}\right) + \tfrac{2}{3}\left(\tfrac{3}{5}\right) &= 2 \\ \tfrac{2y}{3} + \tfrac{6}{15} &= 2 \end{aligned}$$

Next, isolate the variable by subtracting $\frac{6}{15}$ from both sides of the equation.

$$\frac{2y}{3} + \frac{6}{15} - \frac{6}{15} = 2 - \frac{6}{15}$$

Next, simplify each side of the equation.

$$\begin{aligned} \tfrac{2y}{3} &= 2 - \tfrac{6}{15} \\ \tfrac{2y}{3} &= \tfrac{2}{1}\left(\tfrac{15}{15}\right) - \tfrac{6}{15} \\ \tfrac{2y}{3} &= \tfrac{30}{15} - \tfrac{6}{15} \\ \tfrac{2y}{3} &= \tfrac{24}{15} \end{aligned}$$

Next, multiply both sides of the equation by 15.

$$\begin{aligned}\left(\cancel{15}^{5}\right)\left(\frac{2y}{\cancel{3}}\right) &= \left(\cancel{15}^{1}\right)\left(\frac{24}{\cancel{15}}\right)\\ 10y &= 24\end{aligned}$$

Then, divide both sides of the equation by 10.

$$\begin{aligned}\frac{\cancel{10}^{1}y}{\cancel{10}} &= \frac{24}{10}\\ y &= 2\frac{4}{10}\\ y &= 2\frac{2}{5}\end{aligned}$$

The answer is $y = 2\frac{2}{5}$.

Examples

Example 1

Earlier, you were given a problem about Kathleen who was walking for "Relay for Life?"

Her brother, Mack, wants to write an equation to figure out how many miles she walked each day.

First, name the variable. Let 'w' represent the number of miles Kathleen walked on Wednesday.

Next, represent "*On Tuesday Kathleen walked one-third as many miles as... on Wednesday*."

$$\frac{1}{3}w$$

Next, represent the information given in the problem using a verbal model.

Verbal Model:

$$(\text{miles walked Mon.}) + (\text{miles walked Tues.}) + (\text{miles walked Wed.}) = (\text{total miles walked})$$

Next, write an equation to represent the verbal model.

$$\underbrace{(\text{miles walked Mon.})}_{4} + \underbrace{(\text{miles walked Tues.})}_{\frac{1}{3}w} + \underbrace{(\text{miles walked Wed.})}_{w} = \underbrace{(\text{total miles walked})}_{24}$$

$$4 + \tfrac{1}{3}w + w = 24$$

Next, solve the equation for the variable.

First, simplify the left side of the equation.

$$\begin{aligned} 4 + \tfrac{1}{3}w + w &= 24 \\ 4 + 1\tfrac{1}{3}w &= 24 \\ 4 + \tfrac{4}{3}w &= 24 \end{aligned}$$

Next, isolate the variable by subtracting 4 from both sides of the equation.

$$\begin{aligned} 4 + \tfrac{4}{3}w &= 24 \\ 4 - 4 + \tfrac{4}{3}w &= 24 - 4 \end{aligned}$$

Next, simplify both sides of the equation.

$$\begin{aligned} 4 - 4 + \tfrac{4}{3}w &= 24 - 4 \\ 4 + \tfrac{4}{3}w &= 24 \\ \tfrac{4}{3}w &= 20 \end{aligned}$$

Next, divide both sides of the equation by $\frac{4}{3}$ to solve for 'w'.

$$\begin{aligned} \tfrac{4}{3}w &= 20 \\ \tfrac{4}{3} \div \tfrac{4}{3}w &= 20 \div \tfrac{4}{3} \\ \cancel{\tfrac{4}{3}} \times \tfrac{3}{4} w &= 20 \times \tfrac{3}{2} \\ w &= \tfrac{60}{4} \\ w &= 15 \end{aligned}$$

The answer is $w = 15$.

Kathleen walked 15 miles on Wednesday.

Then, use the answer to figure the number of miles Kathleen walked on Tuesday.

$$\begin{aligned} w &= 15 \\ \textbf{miles walked on Tuesday} &= \tfrac{1}{3}w \\ \tfrac{1}{3}(15) &= 5 \end{aligned}$$

The answer is 5.

Kathleen walked 5 miles on Tuesday.

Example 2

Solve the following equation for the variable:

$$\frac{1}{3} + \frac{4}{5} - n = \frac{2}{15}$$

First, add the fractions on the left side of the equation. Remember the fractions must have a common denominator.

$$\begin{aligned}\frac{1}{3}+\frac{4}{5}-n &= \frac{2}{15}\\ \frac{1}{3}\left(\frac{5}{5}\right)+\frac{4}{5}\left(\frac{3}{3}\right)-n &= \frac{2}{15}\\ \frac{5}{15}+\frac{12}{15}-n &= \frac{2}{15}\\ \frac{17}{15}-n &= \frac{2}{15}\end{aligned}$$

Next, isolate the variable by subtracting $\frac{17}{15}$ from both sides of the equation.

$$\begin{aligned}\frac{17}{15}-n &= \frac{2}{15}\\ \frac{17}{15}-\frac{17}{15}-n &= \frac{2}{15}-\frac{17}{15}\end{aligned}$$

Next, simplify both sides of the equation.

$$\begin{aligned}\frac{17}{15}-\frac{17}{15}-n &= \frac{2}{15}-\frac{17}{15}\\ -n &= \frac{-15}{15}\\ -n &= -1\end{aligned}$$

Then, divide both sides of the equation by -1 to solve for 'n'. When you solve an equation you are finding the value of $+1n$.

$$\begin{aligned}\frac{\cancel{-1}^{1}n}{\cancel{-1}} &= \frac{\cancel{-1}^{1}}{\cancel{-1}}\\ n &= 1\end{aligned}$$

The answer is $n=1$.

Example 3

Solve the following equation for the variable:

$$\frac{3}{6}-\frac{1}{3}+x=1\frac{1}{2}$$

First, subtract the fractions on the left side of the equation. Remember the fractions must have a common denominator.

$$\begin{aligned} \tfrac{3}{6} - \tfrac{1}{3} + x &= 1\tfrac{1}{2} \\ \tfrac{3}{6} - \tfrac{1}{3}\left(\tfrac{2}{2}\right) + x &= 1\tfrac{1}{2} \\ \tfrac{3}{6} - \tfrac{2}{6} + x &= 1\tfrac{1}{2} \\ \tfrac{1}{6} + x &= 1\tfrac{1}{2} \end{aligned}$$

Next, express the mixed number $1\frac{1}{2}$ as an improper fraction.

$$\frac{1}{6} + x = \frac{3}{2}$$

Next isolate the variable by subtracting $\frac{1}{6}$ from both sides of the equation.

$$\begin{aligned} \tfrac{1}{6} + x &= \tfrac{3}{2} \\ \tfrac{1}{6} - \tfrac{1}{6} + x &= \tfrac{3}{2} - \tfrac{1}{6} \end{aligned}$$

Next, simplify both sides of the equation.

$$\begin{aligned} \tfrac{1}{6} - \tfrac{1}{6} + x &= \tfrac{3}{2} - \tfrac{1}{6} \\ x &= \tfrac{3}{2}\left(\tfrac{3}{3}\right) - \tfrac{1}{6} \\ x &= \tfrac{9}{6} - \tfrac{1}{6} \\ x &= \tfrac{8}{6} \end{aligned}$$

Then, express the answer in simplest form.

$$\begin{aligned} x &= 1\tfrac{2}{6} \\ x &= 1\tfrac{1}{3} \end{aligned}$$

The answer is $x = 1\frac{1}{3}$.

Example 4

Solve the following equation for the variable:

$$\frac{2}{5}(m+4) = 6$$

First, apply the distributive property to clear the parenthesis.

$$\begin{aligned} \tfrac{2}{5}(m+4) &= 6 \\ \left(\tfrac{2}{5} \cdot \tfrac{m}{1} + \tfrac{2}{5} \cdot \tfrac{4}{1}\right) &= 6 \\ \tfrac{2m}{5} + \tfrac{8}{5} &= 6 \end{aligned}$$

Next, isolate the variable by subtracting $\frac{8}{5}$ from both sides of the equation.

$$\begin{aligned} \tfrac{2m}{5} + \tfrac{8}{5} &= 6 \\ \tfrac{2m}{5} + \tfrac{8}{5} - \tfrac{8}{5} &= 6 - \tfrac{8}{5} \end{aligned}$$

Next, simplify both sides of the equation.

$$\begin{aligned} \tfrac{2m}{5} + \tfrac{8}{5} - \tfrac{8}{5} &= 6 - \tfrac{8}{5} \\ \tfrac{2m}{5} &= \tfrac{6}{1}\left(\tfrac{5}{5}\right) - \tfrac{8}{5} \\ \tfrac{2m}{5} &= \tfrac{30}{5} - \tfrac{8}{5} \\ \tfrac{2m}{5} &= \tfrac{22}{5} \end{aligned}$$

Next, multiply both sides of the equation by 5.

$$\begin{aligned} \frac{2m}{5} &= \frac{22}{5} \\ \cancel{5}^{1}\left(\frac{2m}{\cancel{5}}\right) &= \cancel{5}^{1}\left(\frac{22}{\cancel{5}}\right) \\ 2m &= 22 \end{aligned}$$

Then, divide both sides of the equation by 2 to solve for 'm'.

$$\begin{aligned} 2m &= 22 \\ \frac{\cancel{2}^{1}m}{\cancel{2}} &= \frac{\cancel{22}^{11}}{\cancel{2}} \\ m &= 1 \end{aligned}$$

The answer is $m = 11$.

Example 5

Solve the following equation for the variable.

$$\frac{3}{4}y - \frac{2}{5} = \frac{1}{2}$$

First, isolate the variable by adding $\frac{2}{5}$ to both sides of the equation.

$$\begin{aligned} \frac{3}{4}y - \frac{2}{5} &= \frac{1}{2} \\ \frac{3}{4}y - \frac{2}{5} + \frac{2}{5} &= \frac{1}{2} + \frac{2}{5} \end{aligned}$$

Next, simplify both sides of the equation.

$$\begin{aligned} \frac{3}{4}y - \frac{2}{5} + \frac{2}{5} &= \frac{1}{2} + \frac{2}{5} \\ \frac{3}{4}y &= \frac{1}{2} \cdot \frac{5}{5} + \frac{2}{5} \cdot \frac{2}{2} \\ \frac{3}{4}y &= \frac{5}{10} + \frac{4}{10} \\ \frac{3}{4}y &= \frac{9}{10} \end{aligned}$$

Next, divide both sides of the equation by $\frac{3}{4}$ to solve for 'y'.

$$\begin{aligned} \frac{3}{4}y &= \frac{9}{10} \\ \frac{3}{4} \div \frac{3}{4}y &= \frac{9}{10} \div \frac{3}{4} \\ \cancel{\frac{3}{4}}^{1} \times \frac{4}{3}y &= \frac{9}{10} \times \frac{4}{3} \\ y &= \frac{36}{30} \\ y &= 1\frac{6}{30} \end{aligned}$$

Then simplify the answer.

$$y = 1\frac{1}{5}$$

The answer is $y = 1\frac{1}{5}$.

Review

Solve the following equations:

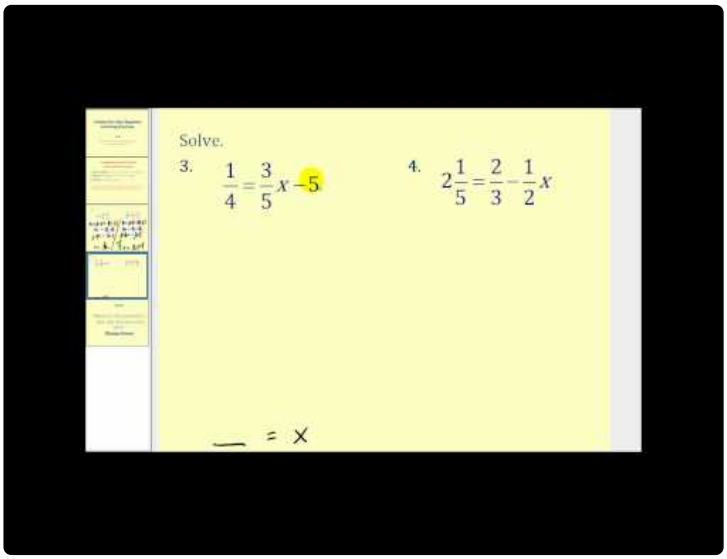
1. $\frac{1}{3}x = 9$
2. $\frac{1}{2}x + \frac{1}{3}x = 10$
3. $\frac{3}{5}y + 1 = 7$
4. $\frac{3}{4}x = 6$
5. $\frac{1}{3} + \frac{4}{6} - x = \frac{1}{2}$
6. $\frac{4}{7} + \frac{2}{7} - x = \frac{2}{7}$
7. $\frac{5}{8}x = 10$
8. $\frac{1}{4}y + 7 = 31$

9. $\frac{1}{3}a - 4 = 12$
10. $\frac{6}{7} - 27 + x = 1\frac{1}{7}$
11. $\frac{4}{5}y - \frac{3}{5}y = 10$
12. $\frac{2}{3}x = 8$
13. $\frac{5}{6} - x = -\frac{1}{6}$
14. $\frac{3}{4}y = \frac{3}{4}$
15. $\frac{6}{8} - \frac{2}{3} + x = \frac{1}{3}$

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

Resources



https://www.ck12.org/flx/render/embeddedobject/167376

Vocabulary

Language: English

Term	Definition
Decimal	In common use, a decimal refers to part of a whole number. The numbers to the left of a decimal point represent whole numbers, and each number to the right of a decimal point represents a fractional part of a power of one-tenth. For instance: The decimal value 1.24 indicates 1 whole unit, 2 tenths, and 4 hundredths (commonly described as 24 hundredths).
Denominator	The denominator of a fraction (rational number) is the number on the bottom and indicates the total number of equal parts in the whole or the group. $\frac{5}{8}$ has denominator 8.
fraction	A fraction is a part of a whole. A fraction is written mathematically as one value on top of another, separated by a fraction bar. It is also called a *rational number*.
Integer	The integers consist of all natural numbers, their opposites, and zero. Integers are numbers in the list ..., -3, -2, -1, 0, 1, 2, 3...
Least Common Denominator	The least common denominator or lowest common denominator of two fractions is the smallest number that is a multiple of both of the original denominators.
rational number	A rational number is a number that can be expressed as the quotient of two integers, with the denominator not equal to zero.
Repeating Decimal	A repeating decimal is a decimal number that ends with a group of digits that repeat indefinitely. 1.666... and 0.9898... are examples of repeating decimals.
Terminating Decimal	A terminating decimal is a decimal number that ends. The decimal number 0.25 is an example of a terminating decimal.

1.0 REFERENCES

Image	Attributions
	**Credit:** DieselDemon **Source:** https://www.flickr.com/photos/28096801@N05/3525799414/in/photolist-6nyDk5-Pwx1w-qTGrtv-qTuKef-rwaGiz-cJeT4-rxUvMJ-cJeNr-cJfkg-cJhif-cJfN5-ry374X-cJhce-cJg2Q-fuydV-cJfuJ-58ZkzV-cJhrD-cJfG4-fdvX9A-fdjGe2-fdgzXK-fdwcgJ-fdgHkZ-fdx6yC-fdiBrz-fdgYUe-fdiMXv-fdwWvN-fdydQS-fdjgwa-fdk4xB-fdhkr6-fdyBGj-fdxzYy-fdiJyP-fdxgNU-fdxahY-fdx3yU-fdyV5b-fdiDBv-fdjqyn-fdibtX-fdwtyG-fdxZtU-fdw8gy-fdiadX-fdiv9g-fdyr3Y-fdyiz5