Exponential Properties Involving Products

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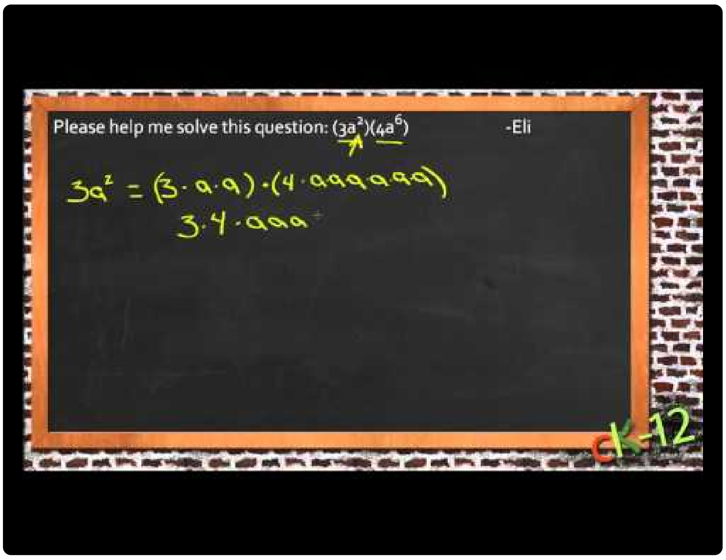
Exponential Properties Involving Products

In expressions involving exponents, like 3^5 or x^3 . the number on the bottom is called the **base** and the number on top is the **power** or **exponent**. The whole expression is equal to the base multiplied by itself a number of times equal to the exponent; in other words, the exponent tells us how many copies of the base number to multiply together.



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Writing Expressions in Exponential Form

Write in exponential form.

a) $2 \cdot 2$

$2 \cdot 2 = 2^2$ because we have 2 factors of 2

b) $(-3)(-3)(-3)$

$(-3)(-3)(-3) = (-3)^3$ because we have 3 factors of (-3)

c) $y \cdot y \cdot y \cdot y \cdot y$

$y \cdot y \cdot y \cdot y \cdot y = y^5$ because we have 5 factors of y

d) $(3a)(3a)(3a)(3a)$

$(3a)(3a)(3a)(3a) = (3a)^4$ because we have 4 factors of $3a$

When the base is a variable, it's convenient to leave the expression in exponential form; if we didn't write x^7, we'd have to write $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$ instead. But when the base is a number, we can simplify the expression further than that; for example, 2^7 equals $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, but we can multiply all those 2's to get 128.

Let's simplify the expressions from Example A.

Simplifying Expressions

Simplify

a) 2^2

$2^2 = 2 \cdot 2 = 4$

b) $(-3)^3$

$(-3)^3 = (-3)(-3)(-3) = -27$

c) y^5

y^5 is already simplified

d) $(3a)^4$

$(3a)^4 = (3a)(3a)(3a)(3a) = 3 \cdot 3 \cdot 3 \cdot 3 \cdot a \cdot a \cdot a \cdot a = 81a^4$

Be careful when taking powers of negative numbers. Remember these rules:

$$\begin{aligned}(negative\ number) \cdot (positive\ number) &= negative\ number \\ (negative\ number) \cdot (negative\ number) &= positive\ number\end{aligned}$$

So **even powers of negative numbers** are always positive. Since there are an even number of factors, we pair up the negative numbers and all the negatives cancel out.

$$\begin{aligned}(-2)^6 &= (-2)(-2)(-2)(-2)(-2)(-2) \\ &= \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)(-2)}_{+4} \\ &= +64\end{aligned}$$

And **odd powers of negative numbers** are always negative. Since there are an odd number of factors, we can still pair up negative numbers to get positive numbers, but there will always be one negative factor left over, so the answer is negative:

$$\begin{aligned}(-2)^5 &= (-2)(-2)(-2)(-2)(-2) \\ &= \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)}_{-2} \\ &= -32\end{aligned}$$

Use the Product of Powers Property

So what happens when we multiply one power of x by another? Let's see what happens when we multiply x ***to the power of 5*** by x ***cubed***. To illustrate better, we'll use the full factored form for each:

$$\underbrace{(x \cdot x \cdot x \cdot x \cdot x)}_{x^5} \cdot \underbrace{(x \cdot x \cdot x)}_{x^3} = \underbrace{(x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x)}_{x^8}$$

So $x^5 \times x^3 = x^8$. You may already see the pattern to multiplying powers, but let's confirm it with another example. We'll multiply x ***squared*** by x ***to the power of 4***:

$$\underbrace{(x \cdot x)}_{x^2} \cdot \underbrace{(x \cdot x \cdot x \cdot x)}_{x^4} = \underbrace{(x \cdot x \cdot x \cdot x \cdot x \cdot x)}_{x^6}$$

So $x^2 \times x^4 = x^6$. Look carefully at the powers and how many factors there are in each calculation. 5 x 's times 3 x 's equals $(5+3) = 8$ x 's. 2 x 's times 4 x 's equals $(2+4) = 6$ x 's.

You should see that when we take the product of two powers of x , the number of x 's in the answer is the total number of x 's in all the terms you are multiplying. In other words, the exponent in the answer is the sum of the exponents in the product.

Product Rule for Exponents: $x^n \cdot x^m = x^{(n+m)}$

There are some easy mistakes you can make with this rule, however. Let's see how to avoid them.

Multiplying Exponents

1. Multiply $2^2 \cdot 2^3$.

$2^2 \cdot 2^3 = 2^5 = 32$

Note that when you use the product rule you **don't multiply the bases**. In other words, you must avoid the common error of writing $2^2 \cdot 2^3 = 4^5$. You can see this is true if you multiply out each expression: 4 times 8 is definitely 32, not 1024.

2. Multiply $2^2 \cdot 3^3$.

$2^2 \cdot 3^3 = 4 \cdot 27 = 108$

In this case, we can't actually use the product rule at all, because it only applies to terms that have the *same base*. In a case like this, where the bases are different, we just have to multiply out the numbers by hand—the answer is *not* 2^5 or 3^5 or 6^5 or anything simple like that.

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Examples

Simplify the following exponents:

Example 1

$(-2)^5$

$(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$

Example 2

$(10x)^2$

$(10x)^2 = 10^2 \cdot x^2 = 100x^2$

Review

Write in exponential notation:

1. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

2. $3x \cdot 3x \cdot 3x$
3. $(-2a)(-2a)(-2a)(-2a)$
4. $6 \cdot 6 \cdot 6 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$
5. $2 \cdot x \cdot y \cdot 2 \cdot 2 \cdot y \cdot x$

Find each number.

6. 5^4
7. $(-2)^6$
8. $(0.1)^5$
9. $(-0.6)^3$
10. $(1.2)^2 + 5^3$
11. $3^2 \cdot (0.2)^3$

Multiply and simplify:

12. $6^3 \cdot 6^6$
13. $2^2 \cdot 2^4 \cdot 2^6$
14. $3^2 \cdot 4^3$
15. $x^2 \cdot x^4$
16. $(-2y^4)(-3y)$
17. $(4a^2)(-3a)(-5a^4)$

Review (Answers)

To see the review answers, return to the Table of Contents and select ‘Other Versions’ or ‘Resources’.

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1.0 REFERENCES

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