

Exponential Properties Involving Products

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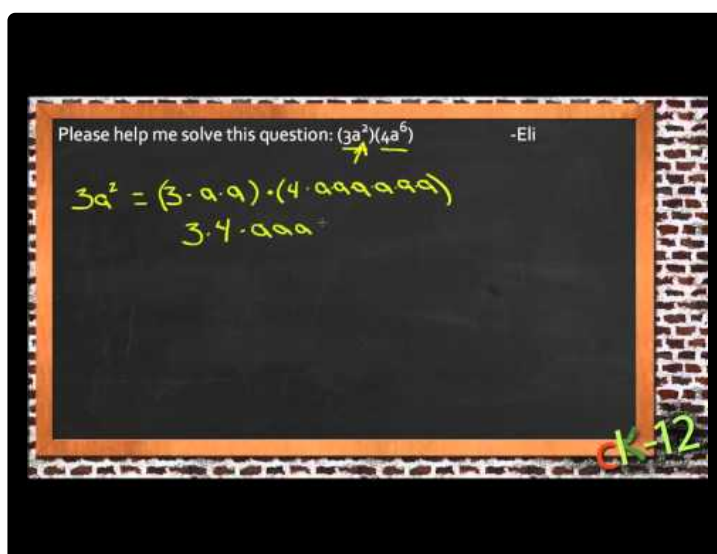
5.1 Exponential Properties Involving Products

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Exponential Properties Involving Products

In expressions involving exponents, like 3^5 or x^3 , the number on the bottom is called the **base** and the number on top is the **power** or **exponent**. The whole **expression** is equal to the base multiplied by itself a number of times equal to the exponent; in other words, the exponent tells us how many copies of the base number to multiply together.



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Writing Expressions in Exponential Form

Write in exponential form.

a) $2 \cdot 2$

$$2 \cdot 2 = 2^2 \text{ because we have 2 factors of 2}$$

b) $(-3)(-3)(-3)$

$$(-3)(-3)(-3) = (-3)^3 \text{ because we have 3 factors of } (-3)$$

c) $y \cdot y \cdot y \cdot y \cdot y$

$$y \cdot y \cdot y \cdot y \cdot y = y^5 \text{ because we have 5 factors of } y$$

d) $(3a)(3a)(3a)(3a)$

$$(3a)(3a)(3a)(3a) = (3a)^4 \text{ because we have 4 factors of } 3a$$

When the base is a **variable**, it's convenient to leave the expression in exponential form; if we didn't write x^7 , we'd have to write $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$ instead. But when the base is a number, we can simplify the expression further than that; for example, 2^7 equals $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, but we can multiply all those 2's to get 128.

Let's simplify the expressions from Example A.

Simplifying Expressions

Simplify

a) 2^2

$$2^2 = 2 \cdot 2 = 4$$

b) $(-3)^3$

$$(-3)^3 = (-3)(-3)(-3) = -27$$

c) y^5

y^5 is already simplified

d) $(3a)^4$

$$(3a)^4 = (3a)(3a)(3a)(3a) = 3 \cdot 3 \cdot 3 \cdot 3 \cdot a \cdot a \cdot a \cdot a = 81a^4$$

Be careful when taking powers of negative numbers. Remember these rules:

$$\begin{aligned}(\textit{negative number}) \cdot (\textit{positive number}) &= \textit{negative number} \\ (\textit{negative number}) \cdot (\textit{negative number}) &= \textit{positive number}\end{aligned}$$

So **even powers of negative numbers** are always positive. Since there are an even number of factors, we pair up the negative numbers and all the negatives cancel out.

$$\begin{aligned}(-2)^6 &= (-2)(-2)(-2)(-2)(-2)(-2) \\ &= \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)(-2)}_{+4} \\ &= +64\end{aligned}$$

And **odd powers of negative numbers** are always negative. Since there are an odd number of factors, we can still pair up negative numbers to get positive numbers, but there will always be one negative **factor** left over, so the answer is negative:

$$\begin{aligned}(-2)^5 &= (-2)(-2)(-2)(-2)(-2) \\ &= \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)}_{-2} \\ &= -32\end{aligned}$$

Use the Product of Powers Property

So what happens when we multiply one power of x by another? Let's see what happens when we multiply x **to the power of 5** by x **cubed**. To illustrate better, we'll use the full **factored form** for each:

$$\underbrace{(x \cdot x \cdot x \cdot x \cdot x)}_{x^5} \cdot \underbrace{(x \cdot x \cdot x)}_{x^3} = \underbrace{(x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x)}_{x^8}$$

So $x^5 \times x^3 = x^8$. You may already see the **pattern** to multiplying powers, but let's confirm it with another example. We'll multiply x **squared** by x **to the power of 4**:

$$\underbrace{(x \cdot x)}_{x^2} \cdot \underbrace{(x \cdot x \cdot x \cdot x)}_{x^4} = \underbrace{(x \cdot x \cdot x \cdot x \cdot x \cdot x)}_{x^6}$$

So $x^2 \times x^4 = x^6$. Look carefully at the powers and how many factors there are in each calculation. **5** x 's times **3** x 's equals **(5 + 3) = 8** x 's. **2** x 's times **4** x 's equals **(2 + 4) = 6** x 's.

You should see that when we take the product of two powers of x , the number of x 's in the answer is the total number of x 's in all the **terms** you are multiplying. In other words, the exponent in the answer is the sum of the exponents in the product.

Product Rule for Exponents: $x^n \cdot x^m = x^{(n+m)}$

There are some easy mistakes you can make with this rule, however. Let's see how to avoid them.

Multiplying Exponents

1. Multiply $2^2 \cdot 2^3$.

$$2^2 \cdot 2^3 = 2^5 = 32$$

Note that when you use the product rule you **don't multiply the bases**. In other words, you must avoid the common error of writing $2^2 \cdot 2^3 = 4^5$. You can see this is true if you multiply out each expression: 4 times 8 is definitely 32, not 1024.

2. Multiply $2^2 \cdot 3^3$.

$$2^2 \cdot 3^3 = 4 \cdot 27 = 108$$

In this case, we can't actually use the product rule at all, because it only applies to terms that have the *same base*. In a case like this, where the bases are different, we just have to multiply out the numbers by hand—the answer is *not* 2^5 or 3^5 or 6^5 or anything simple like that.



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Examples

Simplify the following exponents:

Example 1

$$(-2)^5$$

$$(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$$

Example 2

$$(10x)^2$$

$$(10x)^2 = 10^2 \cdot x^2 = 100x^2$$

Review

Write in exponential notation:

1. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

2. $3x \cdot 3x \cdot 3x$

3. $(-2a)(-2a)(-2a)(-2a)$

4. $6 \cdot 6 \cdot 6 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$

5. $2 \cdot x \cdot y \cdot 2 \cdot 2 \cdot y \cdot x$

Find each number.

6. 5^4

7. $(-2)^6$

8. $(0.1)^5$

9. $(-0.6)^3$

10. $(1.2)^2 + 5^3$

11. $3^2 \cdot (0.2)^3$

Multiply and simplify:

12. $6^3 \cdot 6^6$

13. $2^2 \cdot 2^4 \cdot 2^6$

14. $3^2 \cdot 4^3$

15. $x^2 \cdot x^4$

16. $(-2y^4)(-3y)$

17. $(4a^2)(-3a)(-5a^4)$

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.



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