Exponential Terms Raised to an Exponent

Andrew Gloag
Eve Rawley
Anne Gloag

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-source, collaborative, and web-based compilation model, CK-12 pioneers and promotes the creation and distribution of high-quality, adaptive online textbooks that can be mixed, modified and printed (i.e., the FlexBook® textbooks).

Printed: December 11, 2023 (PST)

AUTHORS
Andrew Gloag
Eve Rawley
Anne Gloag

5.2 Exponential Terms Raised to an Exponent

FlexBooks 2.0 > VUB Math > Exponential Terms Raised to an Exponent

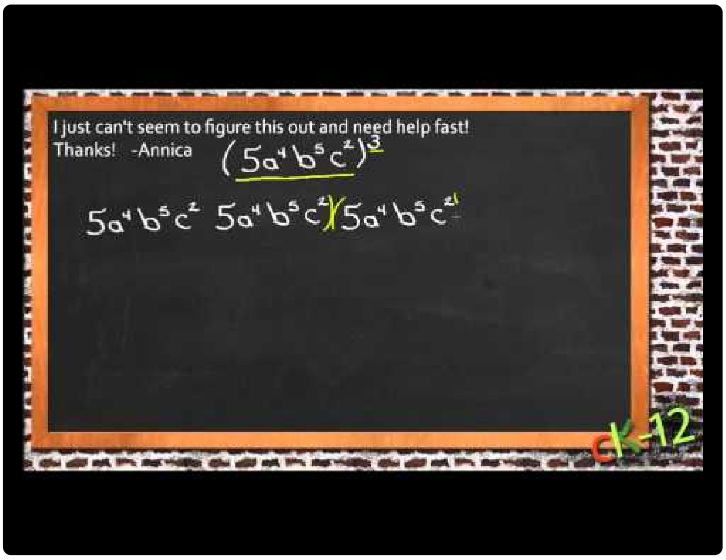
Last Modified: Jan 06, 2023

Exponential Terms Raised to an Exponent

What happens when we raise a whole expression to a power? Let's take x ***to the power of 4*** and ***cube it***. Again we'll use the full factored form for each expression:

$$(x^4)^3 = x^4 \times x^4 \times x^4 \qquad 3\ factors\ of\ \{x\ to\ the\ power\ 4\}$$
$$(x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x) = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^{12}$$

So $(x^4)^3 = x^{12}$. You can see that when we raise a power of x to a new power, the powers multiply.



https://flexbooks.ck12.org/flx/render/embeddedobject/133164

Power Rule for Exponents: $(x^n)^m = x^{(n \cdot m)}$

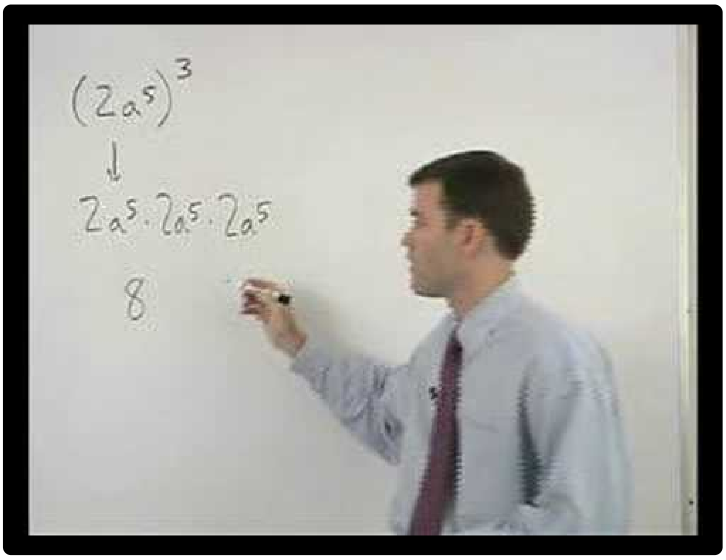
If we have a product of more than one term inside the parentheses, then we have to distribute the exponent over all the factors, like distributing multiplication over addition. For example:

$$(x^2y)^4 = (x^2)^4 \cdot (y)^4 = x^8y^4.$$

Or, writing it out the long way:

$$\begin{aligned}(x^2y)^4 = (x^2y)(x^2y)(x^2y)(x^2y) &= (x \cdot x \cdot y)(x \cdot x \cdot y)(x \cdot x \cdot y)(x \cdot x \cdot y) \\ &= x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y = x^8y^4\end{aligned}$$

Note that this does NOT work if you have a sum or difference inside the parentheses! For example, $(x+y)^2 \neq x^2+y^2$. This is an easy mistake to make, but you can avoid it if you remember what an exponent means: if you multiply out $(x+y)^2$ it becomes $(x+y)(x+y)$, and that's not the same as x^2+y^2 . We'll learn how we can simplify this expression in a later chapter.



https://flexbooks.ck12.org/flx/render/embeddedobject/21510

Simplifying Expressions

1. Simplify the following expressions.

When we're just working with numbers instead of variables, we can use the product rule and the power rule, or we can just do the multiplication and then simplify.

a) $3^5 \cdot 3^7$

We can use the product rule first and then evaluate the result: $3^5 \cdot 3^7 = 3^{12} = 531441$.

OR we can evaluate each part separately and then multiply them: $3^5 \cdot 3^7 = 243 \cdot 2187 = 531441$.

b) $2^6 \cdot 2$

We can use the product rule first and then evaluate the result: $2^6 \cdot 2 = 2^7 = 128$.

OR we can evaluate each part separately and then multiply them: $2^6 \cdot 2 = 64 \cdot 2 = 128$.

c) $(4^2)^3$

We can use the power rule first and then evaluate the result: $(4^2)^3 = 4^6 = 4096$.

OR we can evaluate the expression inside the parentheses first, and then apply the exponent outside the parentheses: $(4^2)^3 = (16)^3 = 4096$.

2. Simplify the following expressions.

When we're just working with variables, all we can do is simplify as much as possible using the product and power rules.

a) $x^2 \cdot x^7$

$x^2 \cdot x^7 = x^{2+7} = x^9$

b) $(y^3)^5$

$(y^3)^5 = y^{3 \times 5} = y^{15}$

3. Simplify the following expressions.

When we have a mix of numbers and variables, we apply the rules to each number and variable separately.

a) $(3x^2y^3) \cdot (4xy^2)$

First we group like terms together: $(3x^2y^3) \cdot (4xy^2) = (3 \cdot 4) \cdot (x^2 \cdot x) \cdot (y^3 \cdot y^2)$

Then we multiply the numbers or apply the product rule on each grouping: $= 12x^3y^5$

b) $(4xyz) \cdot (x^2y^3) \cdot (2yz^4)$

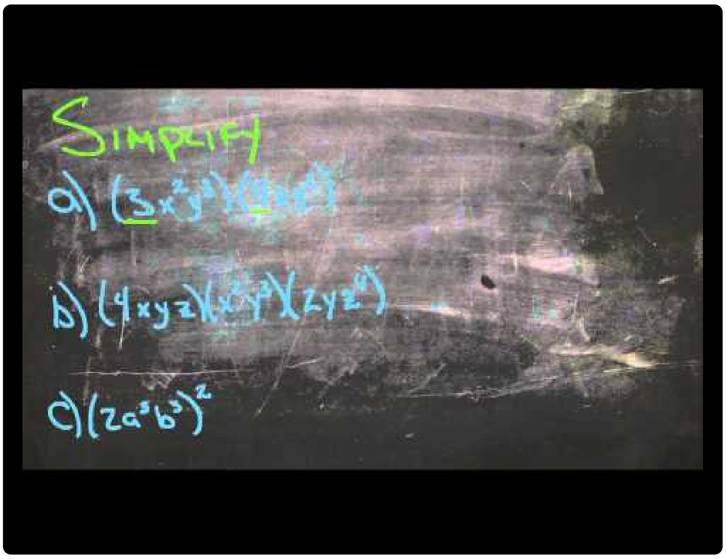
Group like terms together:
$(4xyz) \cdot (x^2y^3) \cdot (2yz^4) = (4 \cdot 2) \cdot (x \cdot x^2) \cdot (y \cdot y^3 \cdot y) \cdot (z \cdot z^4)$

Multiply the numbers or apply the product rule on each grouping: $= 8x^3y^5z^5$

c) $(2a^3b^3)^2$

Apply the power rule for each separate term in the parentheses:
$(2a^3b^3)^2 = 2^2 \cdot (a^3)^2 \cdot (b^3)^2$

Multiply the numbers or apply the power rule for each term $= 4a^6b^6$



https://flexbooks.ck12.org/flx/render/embeddedobject/133165

Examples

Simplify the following expressions.

In problems where we need to apply the product and power rules together, we must keep in mind the order of operations. Exponent operations take precedence over multiplication.

Example 1

$(x^2)^2 \cdot x^3$

We apply the power rule first: $(x^2)^2 \cdot x^3 = x^4 \cdot x^3$

Then apply the product rule to combine the two terms : $x^4 \cdot x^3 = x^7$

Example 2

$(2x^2y) \cdot (3xy^2)^3$

Apply the power rule first: $(2x^2y) \cdot (3xy^2)^3 = (2x^2y) \cdot (27x^3y^6)$

Then apply the product rule to combine the two terms: $(2x^2y) \cdot (27x^3y^6) = 54x^5y^7$

Example 2

$(4a^2b^3)^2 \cdot (2ab^4)^3$

Apply the power rule on each of the terms separately:
$(4a^2b^3)^2 \cdot (2ab^4)^3 = (16a^4b^6) \cdot (8a^3b^{12})$

Then apply the product rule to combine the two terms: $(16a^4b^6) \cdot (8a^3b^{12}) = 128a^7b^{18}$

Review

Simplify:

1. $(a^3)^4$
2. $(xy)^2$
3. $(-5y)^3$
4. $(3a^2b^3)^4$
5. $(-2xy^4z^2)^5$
6. $(-8x)^3(5x)^2$
7. $(-x)^2(xy)^3$
8. $(4a^2)(-2a^3)^4$
9. $(12xy)(12xy)^2$
10. $(2xy^2)(-x^2y)^2(3x^2y^2)$

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

Report Content Errors

1.0 REFERENCES

Image	Attributions