# Exponential Properties Involving Quotients

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Printed: December 11, 2023 (PST)


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# 5.3 Exponential Properties Involving Quotients



Last Modified: Jan 06, 2023

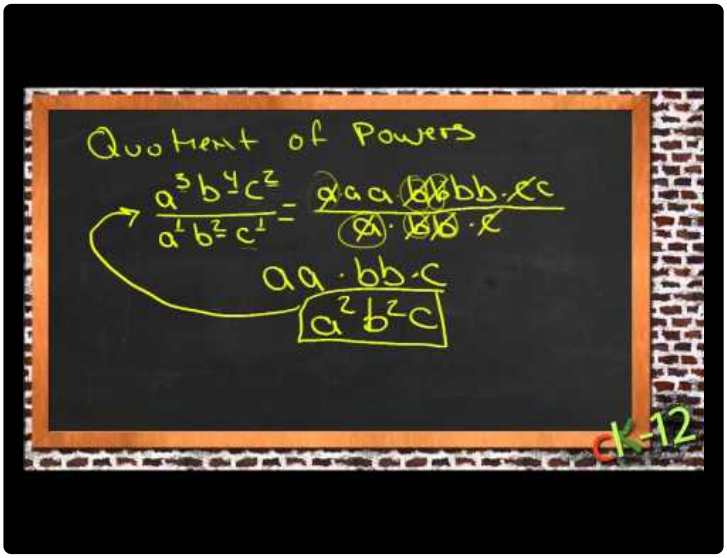
## Exponential Properties Involving Quotients

The rules for simplifying quotients of exponents are a lot like the rules for simplifying products.

Let's look at what happens when we divide $x^7$ by $x^4$:

$$\frac{x^7}{x^4} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = \frac{x \cdot x \cdot x}{1} = x^3$$

You can see that when we divide two powers of $x$, the number of $x$'s in the solution is the number of $x$'s in the top of the fraction minus the number of $x$'s in the bottom. In other words, when dividing expressions with the same base, we keep the same base and simply subtract the exponent in the denominator from the exponent in the numerator.

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**Quotient Rule for Exponents:** $\frac{x^n}{x^m} = x^{(n-m)}$

When we have expressions with more than one base, we apply the quotient rule separately for each base:

Now let's see what happens if the exponent in the denominator is bigger than the exponent in the numerator. For example, what happens when we apply the quotient rule to $\frac{x^4}{x^7}$ ?

The quotient rule tells us to subtract the exponents. 4 minus 7 is -3, so our answer is $x^{-3}$ . A negative exponent! What does that mean?

$$\frac{x^5y^3}{x^3y^2} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} \cdot \frac{\cancel{y} \cdot \cancel{y} \cdot y}{\cancel{y} \cdot \cancel{y}} = \frac{x \cdot x}{1} \cdot \frac{y}{1} = x^2y$$

OR

$$\frac{x^5y^3}{x^3y^2} = x^{5-3} \cdot y^{3-2} = x^2y$$

Well, let's look at what we get when we do the division longhand by writing each term in factored form:

$$\frac{x^4}{x^7} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x} = \frac{1}{x \cdot x \cdot x} = \frac{1}{x^3}$$

Even when the exponent in the denominator is bigger than the exponent in the numerator, we can still subtract the powers. The $x$ 's that are left over after the others have been canceled out just end up in the denominator instead of the numerator. Just as $\frac{x^7}{x^4}$ would be equal to $\frac{x^3}{1}$ (or simply $x^3$ ), $\frac{x^4}{x^7}$ is equal to $\frac{1}{x^3}$ . And you can also see that $\frac{1}{x^3}$ is equal to $x^{-3}$ . We'll learn more about negative exponents shortly.

## Simplifying Expressions

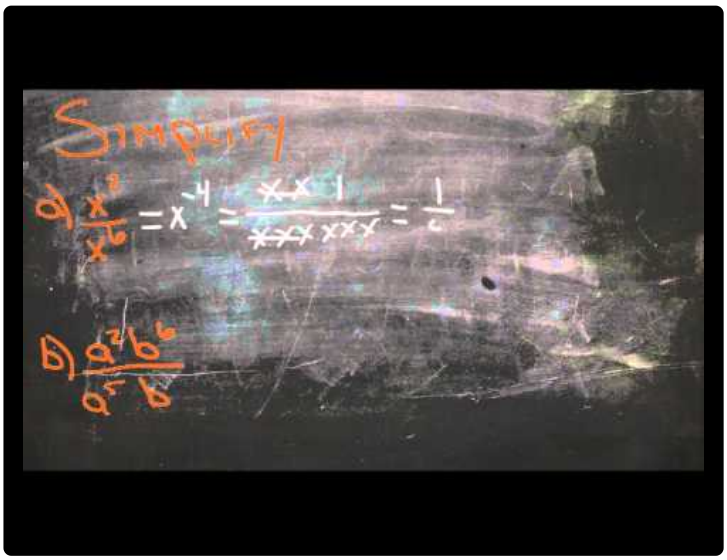
Simplify the following expressions, leaving all exponents positive.

a) $\frac{x^2}{x^6}$

Subtract the exponent in the numerator from the exponent in the denominator and leave the $x$'s in the denominator: $\frac{x^2}{x^6} = \frac{1}{x^{6-2}} = \frac{1}{x^4}$

b) $\frac{a^2b^6}{a^5b}$

Apply the rule to each variable separately: $\frac{a^2b^6}{a^5b} = \frac{1}{a^{5-2}} \cdot \frac{b^{6-1}}{1} = \frac{b^5}{a^3}$

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## Examples

Simplify each of the following expressions using the quotient rule.

### Example 1

$\frac{x^{10}}{x^5}$

$\frac{x^{10}}{x^5} = x^{10-5} = x^5$

### Example 2

$$\frac{a^6}{a}$$

$$\frac{a^6}{a} = a^{6-1} = a^5$$

## Example 3

$$\frac{a^5 b^4}{a^3 b^2}$$

c) $\frac{a^5 b^4}{a^3 b^2} = a^{5-3} \cdot b^{4-2} = a^2 b^2$

## Review

Evaluate the following expressions.

1. $\frac{5^6}{5^2}$
2. $\frac{6^7}{6^3}$
3. $\frac{3^4}{3^{10}}$
4. $\frac{2^2 \cdot 3^2}{5^2}$
5. $\frac{3^3 \cdot 5^2}{3^7}$

Simplify the following expressions.

6. $\frac{a^3}{a^2}$
7. $\frac{x^5}{x^9}$
8. $\frac{x^6 y^2}{x^2 y^5}$
9. $\frac{6a^3}{2a^2}$
10. $\frac{15x^5}{5x}$

11. $\frac{25yx^6}{20y^5x^2}$

## Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

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