Exponent of a Quotient

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5.4 Exponent of a Quotient

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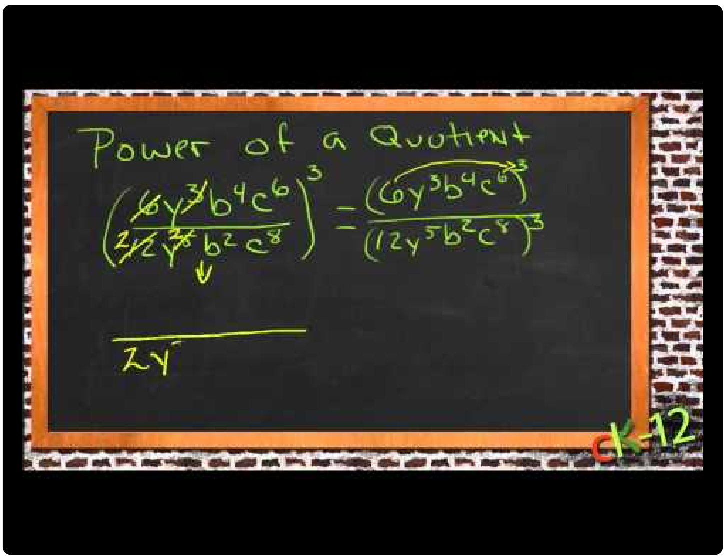
Exponent of a Quotient

When we raise a whole quotient to a power, another special rule applies. Here is an example:

$$\begin{aligned}\left(\frac{x^3}{y^2}\right)^4 &= \left(\frac{x^3}{y^2}\right)\cdot\left(\frac{x^3}{y^2}\right)\cdot\left(\frac{x^3}{y^2}\right)\cdot\left(\frac{x^3}{y^2}\right)\\ &= \frac{(x\cdot x\cdot x)\cdot(x\cdot x\cdot x)\cdot(x\cdot x\cdot x)\cdot(x\cdot x\cdot x)}{(y\cdot y)\cdot(y\cdot y)\cdot(y\cdot y)\cdot(y\cdot y)}\\ &= \frac{x^{12}}{y^8}\end{aligned}$$

Notice that the exponent outside the parentheses is multiplied by the exponent in the numerator and the exponent in the denominator, separately. This is called the power of a quotient rule:

Power Rule for Quotients: $\left(\frac{x^n}{y^m}\right)^p = \frac{x^{n\cdot p}}{y^{m\cdot p}}$



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Let's apply these new rules to a few examples.

Simplifying Expressions

1. Simplify the following expressions.

a) $\frac{4^5}{4^2}$

We can use the quotient rule first and then evaluate the result: $\frac{4^5}{4^2} = 4^{5-2} = 4^3 = 64$

OR we can evaluate each part separately and then divide: $\frac{4^5}{4^2} = \frac{1024}{16} = 64$

b) $\frac{5^3}{5^7}$

Use the quotient rule first and then evaluate the result: $\frac{5^3}{5^7} = \frac{1}{5^4} = \frac{1}{625}$

OR evaluate each part separately and then reduce: $\frac{5^3}{5^7} = \frac{125}{78125} = \frac{1}{625}$

Notice that it makes more sense to apply the quotient rule first for examples (a) and (b). Applying the exponent rules to simplify the expression *before* plugging in actual numbers means that we end up with smaller, easier numbers to work with.

c) $\left(\frac{3^4}{5^2}\right)^2$

Use the power rule for quotients first and then evaluate the result: $\left(\frac{3^4}{5^2}\right)^2 = \frac{3^8}{5^4} = \frac{6561}{625}$

OR evaluate inside the parentheses first and then apply the exponent:
$\left(\frac{3^4}{5^2}\right)^2 = \left(\frac{81}{25}\right)^2 = \frac{6561}{625}$

2. Simplify the following expressions:

a) $\frac{x^{12}}{x^5}$

b) $\left(\frac{x^4}{x}\right)^5$

Use the power rule for quotients and then the quotient rule: $\left(\frac{x^4}{x}\right)^5 = \frac{x^{20}}{x^5} = x^{15}$

OR use the quotient rule inside the parentheses first, then apply the power rule: $\left(\frac{x^4}{x}\right)^5 = (x^3)^5 = x^{15}$

3. Simplify the following expressions.

When we have a mix of numbers and variables, we apply the rules to each number or each variable separately.

a) $\frac{6x^2y^3}{2xy^2}$

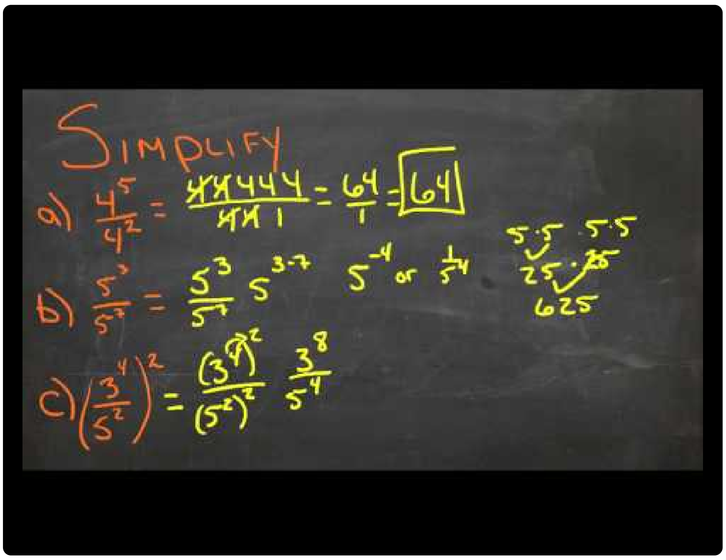
Group like terms together: $\frac{6x^2y^3}{2xy^2} = \frac{6}{2} \cdot \frac{x^2}{x} \cdot \frac{y^3}{y^2}$

Then reduce the numbers and apply the quotient rule on each fraction to get $3xy$.

b) $\left(\frac{2a^3b^3}{8a^7b}\right)^2$

Apply the quotient rule inside the parentheses first: $\left(\frac{2a^3b^3}{8a^7b}\right)^2 = \left(\frac{b^2}{4a^4}\right)^2$

Then apply the power rule for quotients: $\left(\frac{b^2}{4a^4}\right)^2 = \frac{b^4}{16a^8}$



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Examples

Simplify the following expressions.

In problems where we need to apply several rules together, we must keep the order of operations in mind.

Example 1

$(x^2)^2 \cdot \frac{x^6}{x^4}$

We apply the power rule first on the first term:

$$(x^2)^2 \cdot \frac{x^6}{x^4} = x^4 \cdot \frac{x^6}{x^4}$$

Then apply the quotient rule to simplify the fraction:

$$x^4 \cdot \frac{x^6}{x^4} = x^4 \cdot x^2$$

And finally simplify with the product rule:

$$x^4 \cdot x^2 = x^6$$

Example 2

$\left(\frac{16a^2}{4b^5}\right)^3 \cdot \frac{b^2}{a^{16}}$

Simplify inside the parentheses by reducing the numbers:

$$\left(\frac{4a^2}{b^5}\right)^3 \cdot \frac{b^2}{a^{16}}$$

Then apply the power rule to the first fraction:

$$\left(\frac{4a^2}{b^5}\right)^3 \cdot \frac{b^2}{a^{16}} = \frac{64a^6}{b^{15}} \cdot \frac{b^2}{a^{16}}$$

Group like terms together:

$$\frac{64a^6}{b^{15}} \cdot \frac{b^2}{a^{16}} = 64 \cdot \frac{a^6}{a^{16}} \cdot \frac{b^2}{b^{15}}$$

And apply the quotient rule to each fraction:

$$64 \cdot \frac{a^6}{a^{16}} \cdot \frac{b^2}{b^{15}} = \frac{64}{a^{10}b^{13}}$$

Review

Evaluate the following expressions.

1. $\left(\frac{3}{8}\right)^2$
2. $\left(\frac{2^2}{3^3}\right)^3$

3. $\left(\frac{2^3 \cdot 4^2}{2^4}\right)^2$

Simplify the following expressions.

4. $\left(\frac{a^3 b^4}{a^2 b}\right)^3$
5. $\left(\frac{18a^4}{15a^{10}}\right)^4$
6. $\left(\frac{x^6 y^2}{x^4 y^4}\right)^3$
7. $\left(\frac{6a^2}{4b^4}\right)^2 \cdot \frac{5b}{3a}$
8. $\frac{(2a^2 bc^2)(6abc^3)}{4ab^2 c}$
9. $\frac{(2a^2 bc^2)(6abc^3)}{4ab^2 c}$ for $a = 2, b = 1,$ and $c = 3$
10. $\left(\frac{3x^2 y}{2z}\right)^3 \cdot \frac{z^2}{x}$ for $x = 1, y = 2,$ and $z = -1$
11. $\frac{2x^3}{xy^2} \cdot \left(\frac{x}{2y}\right)^2$ for $x = 2, y = -3$
12. $\frac{2x^3}{xy^2} \cdot \left(\frac{x}{2y}\right)^2$ for $x = 0, y = 6$
13. If $a = 2$ and $b = 3$, simplify $\frac{(a^2 b)(bc)^3}{a^3 c^2}$ as much as possible.

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

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