Negative Exponents

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5.5 Negative Exponents

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Negative Exponents

The product and quotient rules for exponents lead to many interesting concepts. For example, so far we've mostly just considered positive, whole numbers as exponents, but you might be wondering what happens when the exponent isn't a positive whole number. What does it mean to raise something to the power of zero, or -1, or $\frac{1}{2}$? In this lesson, we'll find out.

Simplify Expressions With Negative Exponents

When we learned the quotient rule for exponents $\left(\frac{x^n}{x^m} = x^{(n-m)}\right)$, we saw that it applies even when the exponent in the denominator is bigger than the one in the numerator. Canceling out the factors in the numerator and denominator leaves the leftover factors in the denominator, and subtracting the exponents leaves a negative number. So negative exponents simply represent fractions with exponents in the denominator. This can be summarized in a rule:

Negative Power Rule for Exponents: $x^{-n} = \frac{1}{x^n}$, where $x \neq 0$

Negative exponents can be applied to products and quotients also. Here's an example of a negative exponent being applied to a product:

$$\begin{aligned} (x^3y)^{-2} &= x^{-6}y^{-2} && \text{using the power rule} \\ x^{-6}y^{-2} &= \frac{1}{x^6} \cdot \frac{1}{y^2} = \frac{1}{x^6y^2} && \text{using the negative power rule separately on each variable} \end{aligned}$$

And here's one applied to a quotient:

$$\left(\frac{a}{b}\right)^{-3} = \frac{a^{-3}}{b^{-3}} \quad \text{using the power rule for quotients}$$

$$\frac{a^{-3}}{b^{-3}} = \frac{a^{-3}}{1} \cdot \frac{1}{b^{-3}} = \frac{1}{a^3} \cdot \frac{b^3}{1} \quad \text{using the negative power rule on each variable separately}$$

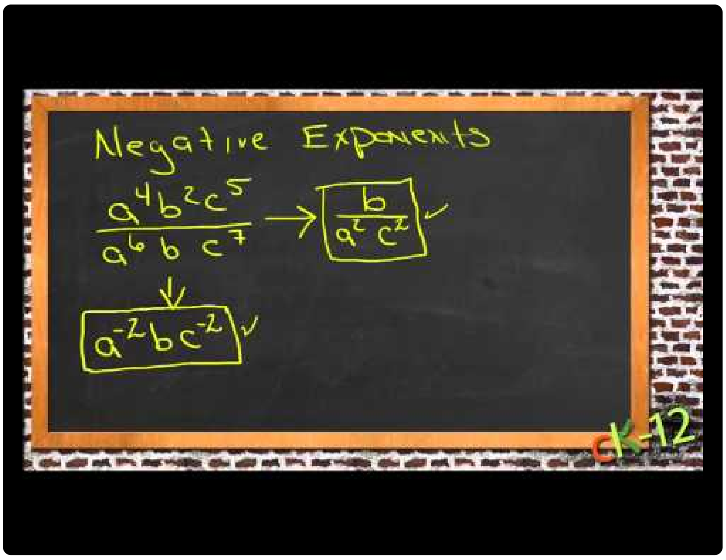
$$\frac{1}{a^3} \cdot \frac{b^3}{1} = \frac{b^3}{a^3} \quad \text{simplifying the division of fractions}$$

$$\frac{b^3}{a^3} = \left(\frac{b}{a}\right)^3 \quad \text{using the power rule for quotients in reverse.}$$

That last step wasn't really necessary, but putting the answer in that form shows us something useful: $\left(\frac{a}{b}\right)^{-3}$ is equal to $\left(\frac{b}{a}\right)^3$. This is an example of a rule we can apply more generally:

Negative Power Rule for Fractions: $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$, where $x \neq 0, y \neq 0$

This rule can be useful when you want to write out an expression without using fractions.



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Writing Expressions Without Fractions

Write the following expressions without fractions.

a) $\frac{1}{x}$

$\frac{1}{x} = x^{-1}$

b) $\frac{2}{x^2}$

$\frac{2}{x^2} = 2x^{-2}$

c) $\frac{x^2}{y^3}$

$\frac{x^2}{y^3} = x^2y^{-3}$

d) $\frac{3}{xy}$

$\frac{3}{xy} = 3x^{-1}y^{-1}$

Simplifying Expressions

Simplify the following expressions and write them without fractions.

a) $\frac{4a^2b^3}{2a^5b}$

Reduce the numbers and apply the quotient rule to each variable separately:

$$\frac{4a^2b^3}{2a^5b} = 2 \cdot a^{2-5} \cdot b^{3-1} = 2a^{-3}b^2$$

b) $\left(\frac{x}{3y^2}\right)^3 \cdot \frac{x^2y}{4}$

Apply the power rule for quotients first:

$$\left(\frac{2x}{y^2}\right)^3 \cdot \frac{x^2y}{4} = \frac{8x^3}{y^6} \cdot \frac{x^2y}{4}$$

Then simplify the numbers, and use the product rule on the x's and the quotient rule on the y's:

$$\frac{8x^3}{y^6} \cdot \frac{x^2y}{4} = 2 \cdot x^{3+2} \cdot y^{1-6} = 2x^5y^{-5}$$

You can also use the negative power rule the other way around if you want to write an expression without negative exponents.

Writing Expressions Without Negative Exponents

Write the following expressions without negative exponents.

a) $3x^{-3}$

$$3x^{-3} = \frac{3}{x^3}$$

b) $a^2b^{-3}c^{-1}$

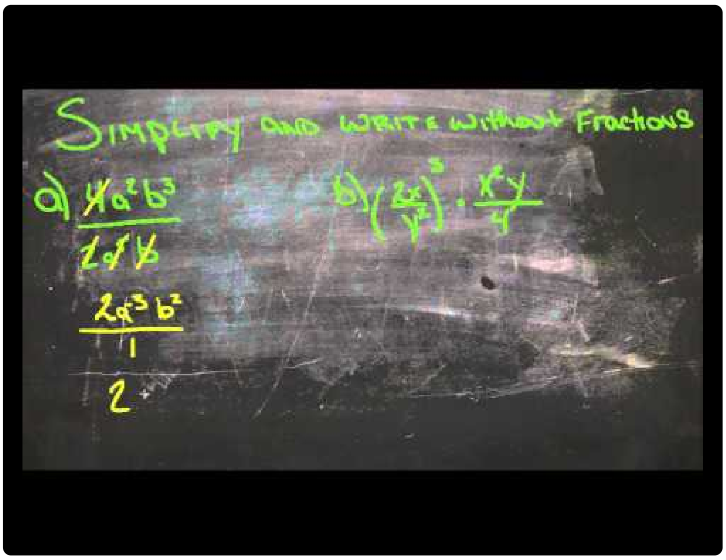
$$a^2b^{-3}c^{-1} = \frac{a^2}{b^3c}$$

c) $4x^{-1}y^3$

$$4x^{-1}y^3 = \frac{4y^3}{x}$$

d) $\frac{2x^{-2}}{y^{-3}}$

$$\frac{2x^{-2}}{y^{-3}} = \frac{2y^3}{x^2}$$



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Examples

Simplify the following expressions and write the answers without negative powers.

Example 1

$$\left(\frac{ab^{-2}}{b^3}\right)^2$$

Apply the quotient rule inside the parentheses: $\left(\frac{ab^{-2}}{b^3}\right)^2 = (ab^{-5})^2$

Then apply the power rule: $(ab^{-5})^2 = a^2b^{-10} = \frac{a^2}{b^{10}}$

Example 2

$$\frac{x^{-3}y^2}{x^2y^{-2}}$$

Apply the quotient rule to each variable separately:
$\frac{x^{-3}y^2}{x^2y^{-2}} = x^{-3-2}y^{2-(-2)} = x^{-5}y^4 = \frac{y^4}{x^5}$

Review

Simplify the following expressions in such a way that there aren't any negative exponents in the answer.

1. $x^{-1}y^{2}$
2. x^{-4}
3. $\frac{x^{-3}}{x^{-7}}$
4. $\frac{x^{-3}y^{-5}}{z^{-7}}$
5. $\left(\frac{a}{b}\right)^{-2}$
6. $(3a^{-2}b^{2}c^{3})^{3}$

Simplify the following expressions in such a way that there aren't any fractions in the answer.

7. $\frac{a^{-3}(a^{5})}{a^{-6}}$
8. $\frac{5x^{6}y^{2}}{x^{8}y}$
9. $\frac{(4ab^{6})^{3}}{(ab)^{5}}$
10. $\frac{(3x^{3})(4x^{4})}{(2y)^{2}}$
11. $\frac{a^{-2}b^{-3}}{c^{-1}}$

Review (Answers)

To see the review answers, return to the Table of Contents and select ‘Other Versions’ or ‘Resources’.

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