Finding Solutions for Equations in Two Variables

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Printed: December 11, 2023 (PST)

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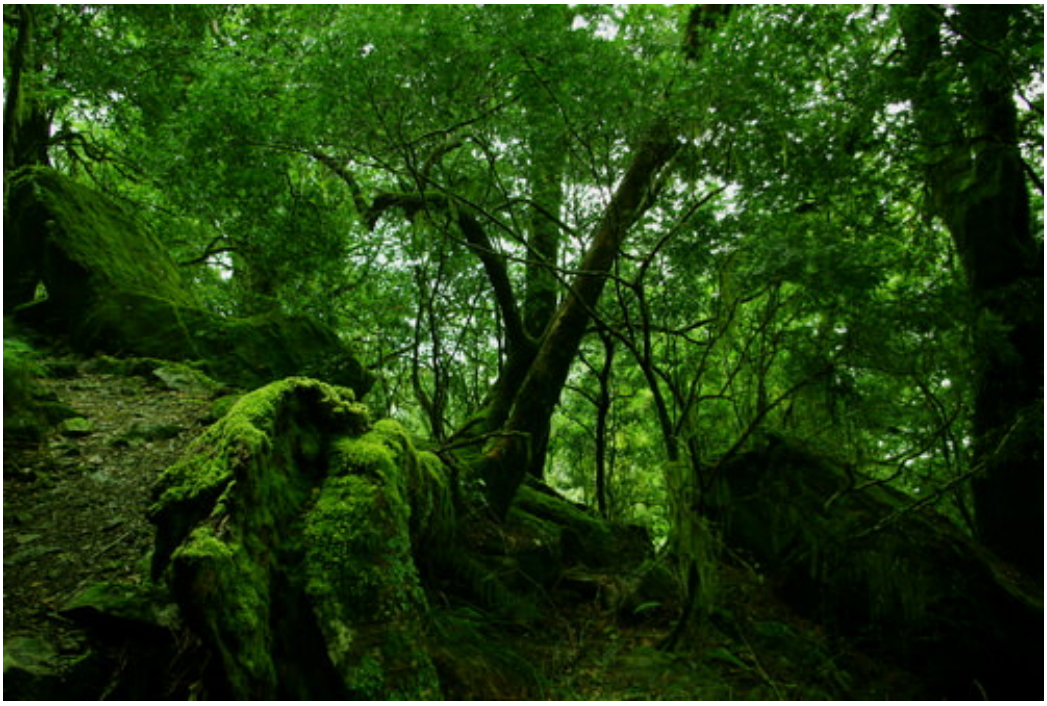
6.3 Finding Solutions for Equations in Two Variables

Last Modified: Aug 23, 2023



[Figure 1]

Mr. Brown, a seventh grade teacher, calls the Science Museum to find out the cost of taking his class to the Omni Theater to see a film on the rainforest. He receives the following information.

The cost of a ticket is $5.00 plus a $2.00 service fee per ticket. There are 22 students in his class and he could have anywhere from 22 to 18 students go based on the number of students absent. Now Mr. Brown has to do the math to figure out the cost of taking the students to see the film.

He knows that he will have to use some type of an equation and a table to record the cost based on the different number of students that may attend.

In this concept, you will learn to find solutions for equations in two variables.

Equations

Function rules can be written using words or symbols. When symbols are used the function rule takes on the form of an equation. Remember, an **equation** is a statement of equality

between two or more quantities. The equation $x + y = 5$ means there are values for 'x' and 'y' that would make the statement true. This equation contains two variables, 'x' and 'y', and any solution can be written as an ordered pair, (x, y).

Let's look at some solutions for the equation.

$$x + y = 5$$

$3 + 2 = 5$ which means $x = 3$ and $y = 2$. The solution can be written as $(3, 2)$.

$7 + (-2) = 5$ which means $x = 7$ and $y = -2$. The solution can be written as $(7, -2)$.

$0 + 5 = 5$ which means $x = 0$ and $y = 5$. The solution can be written as $(0, 5)$.

The number of ordered pairs that will make the given equation a true statement of equality is endless.

Look at the following equation with two variables 'x' and 'y.'

$$2x + y = 12$$

This is another equation that has many ordered pairs that make it a true statement. Write four ordered pairs for the given equation.

$$\begin{aligned} 2x + y &= 12 \\ 2(1) + (10) &= 12 \\ 2 + 10 &= 12 \\ 12 &= 12 \end{aligned}$$ **The ordered pair is $(1, 10)$.**

$$\begin{aligned} 2x + y &= 12 \\ 2(2) + (8) &= 12 \\ 4 + 8 &= 12 \\ 12 &= 12 \end{aligned}$$ **The ordered pair is $(2, 8)$.**

$$\begin{aligned} 2x+y &= 12 \\ 2(3)+(6) &= 12 \\ 6+6 &= 12 \\ 12 &= 12 \end{aligned} \qquad \text{The ordered pair is } (3,6).$$

$$\begin{aligned} 2x+y &= 12 \\ 2(5)+(2) &= 12 \\ 10+2 &= 12 \\ 12 &= 12 \end{aligned} \qquad \text{The ordered pair is } (5,2).$$

The given equation is written in the form $Ax+By=C$ where A and B are coefficients of the variables x and y respectively and C is a constant. An equation written in the form $Ax+By=C$ is said to be written in **standard form**. When an equation is written in standard form, the coefficient 'A' of the variable 'x' must be a positive value.

Another way to write the given equation with two variables 'x' and 'y' is to write it in **function form** as $y=-2x+12$. The variable 'y' has been written as a function of $-2x+12$ which means that the value of the output number 'y' will depend upon the value substituted into the function for 'x' variable. If function notation is used such that $y=f(x)$ then the function form of the equation will be $f(x)=-2x+12$. Whatever form is used, the value of the output number will change as the value of the input number changes.

Use the given equation, written in function form, to complete the following table.

$$y=3x-5$$

x	y
1	-2
2	1
3	4
4	7
5	10

First, substitute '1' into the function for the variable 'x.'

$$\begin{aligned} y &= 3x - 5 \\ y &= 3(1) - 5 \end{aligned}$$

Next, perform the multiplication to clear the parenthesis.

$$\begin{aligned} y &= 3(1) - 5 \\ y &= 3 - 5 \end{aligned}$$

Next, perform the subtraction on the right side of the equation.

$$\begin{aligned} y &= 3 - 5 \\ y &= -2 \end{aligned}$$

Then, write the answer in the table.

The answer is -2.

Repeat the above process for the remaining values of the variable 'x.'

$$\begin{aligned} x &= 2 \\ y &= 3x - 5 \\ y &= 3(2) - 5 \\ y &= 6 - 5 \\ y &= 1 \end{aligned}$$

The answer is 1.

$$\begin{aligned} x &= 3 \\ y &= 3x - 5 \\ y &= 3(3) - 5 \\ y &= 9 - 5 \\ y &= 4 \end{aligned}$$

The answer is 4.

$$\begin{aligned} x &= 4 \\ y &= 3x - 5 \\ y &= 3(4) - 5 \\ y &= 12 - 5 \\ y &= 7 \end{aligned}$$

The answer is 7.

$$\begin{aligned} x &= 5 \\ y &= 3x - 5 \\ y &= 3(5) - 5 \\ y &= 15 - 5 \\ y &= 10 \end{aligned}$$

The answer is 10.

Examples

Example 1

Earlier, you were given a problem about Mr. Brown and the rainforest movie. He needs to figure out the cost of taking the students to see the rainforest movie. He can write an equation in function form to represent the information he received from the Science Museum.

First, write the information from the Science Museum.

Each ticket costs $5.00 plus a $2.00 fee per ticket.

Next, write an equation in function form to represent the information.

Let 'x' represent the number of tickets being purchased and let 'y' represent the total cost for the number of tickets purchased.

$$y = 5x + 2$$

Next, draw a table to record the cost for taking 18, 19, 20, 21 or 22 students to the movie. Each student must purchase a ticket.

Number of Tickets (x)	Total Cost (y)
18	$92.00
19	$97.00
20	$102.00
21	$107.00
22	$112.00

Use the equation to calculate the cost for each group of students to see the movie.

$$\begin{aligned} x &= 18 \\ y &= 5x + 2 \\ y &= 5(18) + 2 && \text{Substitute } x = 18 \text{ into the equation.} \\ y &= 90 + 2 && \text{Perform the multiplication to clear the parenthesis.} \\ y &= \$92.00 && \text{Simplify the right side of the equation.} \end{aligned}$$

Repeat the process above to calculate the cost for the remaining 'x' values.

$$\begin{aligned} x &= 19 \\ y &= 5x + 2 \\ y &= 5(19) + 2 \\ y &= 95 + 2 \\ y &= \$97.00 \end{aligned}$$

$$\begin{aligned} x &= 20 \\ y &= 5x + 2 \\ y &= 5(20) + 2 \\ y &= 100 + 2 \\ y &= \$102.00 \end{aligned}$$

$$\begin{aligned} x &= 21 \\ y &= 5x + 2 \\ y &= 5(21) + 2 \\ y &= 105 + 2 \\ y &= \$107.00 \end{aligned}$$

$$\begin{aligned} x &= 22 \\ y &= 5x + 2 \\ y &= 5(22) + 2 \\ y &= 110 + 2 \\ y &= \$112.00 \end{aligned}$$

Example 2

Write the following equation in standard form.

$$7y - 9 = 4x$$

First, write the standard form of an equation.

$$Ax + By = C$$

Next, add 9 to both sides of the equation and simplify the equation.

$$\begin{aligned} 7y - 9 &= 4x \\ 7y - 9 + 9 &= 4x + 9 \\ 7y &= 4x + 9 \end{aligned}$$

Next, subtract $4x$ from both sides of the equation and simplify the equation.

$$\begin{aligned} 7y &= 4x + 9 \\ 7y - 4x &= 4x - 4x + 9 \\ 7y - 4x &= 9 \end{aligned}$$

Next, rewrite the equation so that its order matches $Ax + By = C$.

$$-4x + 7y = 9$$

Next, multiply both sides of the equation by -1 to change the coefficient of the 'x' variable to a positive value. Simplify the equation.

$$\begin{aligned} -4x + 7y &= 9 \\ -1(-4x + 7y) &= -1(9) \\ 4x - 7y &= -9 \end{aligned}$$

The answer is $4x - 7y = -9$.

Example 3

Write the following equation in function form.

$$4x - 2y = 16$$

Remember for the equation to be expressed in function form it must be solved for the variable 'y.'

First, subtract $4x$ from both sides of the equation and simplify.

$$\begin{aligned} 4x - 2y &= 16 \\ 4x - 4x - 2y &= 16 - 4x \\ -2y &= -4x + 16 \end{aligned}$$

Next, divide each term of the equation by -2 to express the equation in function form.

$$\begin{aligned} -2y &= -4x + 16 \\ \frac{\overset{1}{\cancel{-2}\,y}}{\cancel{-2}} &= \frac{\overset{2}{\cancel{-4}\,x}}{\cancel{-2}} + \frac{\overset{-8}{\cancel{16}}}{\cancel{-2}} \\ y &= 2x - 8 \end{aligned}$$

The answer is $y = 2x - 8$.

Example 4

Write the following equation in standard form.

$$y + 8 = 6x$$

First, write the standard form of an equation.

$$Ax + By = C$$

Next, subtract 8 from both sides of the equation and simplify the equation.

$$\begin{aligned} y + 8 &= 6x \\ y + 8 - 8 &= 6x - 8 \\ y &= 6x - 8 \end{aligned}$$

Next, subtract $6x$ from both sides of the equation and simplify the equation.

$$\begin{aligned} y &= 6x - 8 \\ y - 6x &= 6x - 6x - 8 \\ y - 6x &= -8 \end{aligned}$$

Next, rewrite the equation so that its order matches $Ax + By = C$.

$$-6x + y = -8$$

Next, multiply both sides of the equation by -1 to change the coefficient of the 'x' variable to a positive value. Simplify the equation.

$$\begin{aligned} -6x + y &= -8 \\ -1(-6x + y) &= -1(-8) \\ 6x - y &= 8 \end{aligned}$$

The answer is $6x - y = 8$.

Example 5

Write the following equation in function form and then use the equation to complete the given table.

$$4x - y - 1 = 0$$

x	y
0	-1
2	7
4	15
6	23

First, subtract $4x$ from both sides of the equation and simplify.

$$\begin{aligned} 4x - y - 1 &= 0 \\ 4x - 4x - y - 1 &= 0 - 4x \\ -y - 1 &= -4x \end{aligned}$$

Next, subtract add 1 to both sides of the equation and simplify.

$$\begin{aligned} -y - 1 &= -4x \\ -y - 1 + 1 &= -4x + 1 \\ -y &= -4x + 1 \end{aligned}$$

Next, divide each term of the equation by -1 to express the equation in function form.

$$\begin{aligned} -y &= -4x + 1 \\ \frac{\overset{1}{\cancel{-}y}}{\cancel{-1}} &= \frac{\overset{4}{\cancel{-}4x}}{\cancel{-1}} + \frac{\overset{-1}{\cancel{-}1}}{\cancel{-1}} \\ y &= 4x - 1 \end{aligned}$$

The answer is $y = 4x - 1$.

Next, use the function form of the equation to calculate the output numbers.

$$
\begin{aligned}
x &= 0 \\
y &= 4x - 1 \\
y &= 4(0) - 1 && \text{Substitute } x = 0 \text{ into the equation.} \\
y &= 0 - 1 && \text{Perform the multiplication to clear the parenthesis.} \\
y &= -1 && \text{Simplify the right side of the equation.}
\end{aligned}
$$

$$
\begin{aligned}
x &= 2 \\
y &= 4x - 1 \\
y &= 4(2) - 1 && \text{Substitute } x = 2 \text{ into the equation.} \\
y &= 8 - 1 && \text{Perform the multiplication to clear the parenthesis.} \\
y &= 7 && \text{Simplify the right side of the equation.}
\end{aligned}
$$

$$
\begin{aligned}
x &= 4 \\
y &= 4x - 1 \\
y &= 4(4) - 1 && \text{Substitute } x = 4 \text{ into the equation.} \\
y &= 16 - 1 && \text{Perform the multiplication to clear the parenthesis.} \\
y &= 15 && \text{Simplify the right side of the equation.}
\end{aligned}
$$

$$
\begin{aligned}
x &= 6 \\
y &= 4x - 1 \\
y &= 4(6) - 1 && \text{Substitute } x = 4 \text{ into the equation.} \\
y &= 24 - 1 && \text{Perform the multiplication to clear the parenthesis.} \\
y &= 23 && \text{Simplify the right side of the equation.}
\end{aligned}
$$

Then, complete the table by entering the calculated values in the table.

Review

Find 4 solutions to the function $3x + y = 24$. Write your answers as ordered pairs.

1.

2.

3.

4.

Find 4 solutions to the function $2x - y = 9$. Write your answers are ordered pairs.

5.

6.

7.

8.

Write each equation in standard form.

9. $y = 2x + 3$

10. $y = -4x + 6$

11. $y = -2x - 4$

12. $y = -5x + 4$

13. $y = -3x - 2$

14. $y = -4x - 6$

15. $y = 6x - 1$

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

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