Finding the Slope of a Line

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6.5 Finding the Slope of a Line

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[Figure 1]

Jacques is at the top of a hill in the Whistler Ski Resort. As he looks around he sees a flag with the coordinates $(98, 60)$ blowing in the wind. Jacques wonders if these coordinates have anything to do with the coordinates $(82, 124)$ he saw on a flag on his way up on the lift. What could these coordinates be telling Jacques about the steepness of the hill?

In this concept, you will learn to find the slope of a line.

Slopes

The graph of a linear function is a straight line. Although the graphs are all straight lines, the slant or steepness of the lines vary from graph to graph. The slant or steepness of a straight line is referred to as the **slope** of the line. The slope of a line indicates the rate at which a change in x (the horizontal coordinates) creates a change in the value of y (the vertical coordinates).

In mathematics, the slope of a line is represented by the letter m as defined by:

$$m = \frac{\Delta y}{\Delta x} \rightarrow \frac{\text{change in } y}{\text{change in } x} \rightarrow \frac{\text{rise}}{\text{run}}$$

The slope of a graph can be found by counting as shown on the graph below.

$$m = \frac{\text{rise}}{\text{run}}$$

First, find two exact points (points on the line that are on the corner of a box on the grid) on the graph.

Next, begin with a point to the left and count horizontally to the right until you are either directly above or directly below the second point.

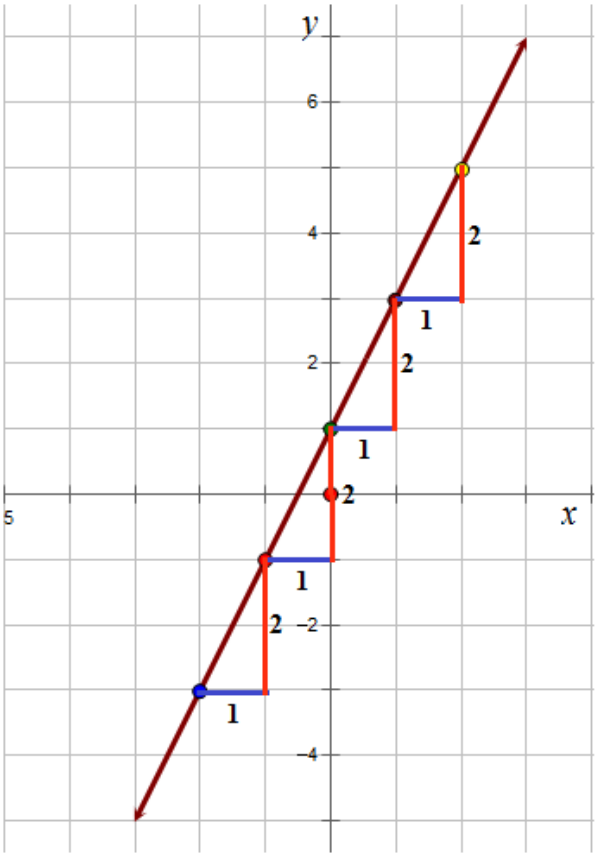
Then, write the number that you have just moved horizontally in a positive direction as the denominator (run) of the slope.

Next, count the number vertically to the second point. If you move vertically upward then the number is positive. If you move vertically downward then the number is negative.

Then, write this value as the numerator (rise) of the slope.

For the following graph:

$$m = \frac{\text{rise}}{\text{run}} \qquad m = \frac{2}{1}$$



[Figure 2]

The slope of a line remains unchanged everywhere on the line. The slope of this line is positive as it moves upward from left to right. If the slope of the line were negative then the line would move downward from left to right.

If you are not given a graph, the slope of a line can be determined by using the coordinates of two points on the line.

Let's look at another example.

The points $A(15, 8)$ and $B(10, 7)$ are on a straight line. What is the slope of the line?

When two points on the straight line are known, the slope can be calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) are the coordinates of the point chosen to be the first point and (x_2, y_2) are the coordinates of the point chosen to be the second point. Of the two given points the one chosen as the first and the one chosen as the second will not affect the value of the slope of the line.

First, choose and label the points as the first point and the second point.

$$\begin{pmatrix} x_1, & y_1 \\ 15, & 8 \end{pmatrix} \text{ and } \begin{pmatrix} x_2, & y_2 \\ 10, & 7 \end{pmatrix}$$

Next, write the formula for finding the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Next, fill in the values for the coordinates as they have been named.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{7 - 8}{10 - 15} \end{aligned}$$

Next, simplify the numerator and the denominator of the formula.

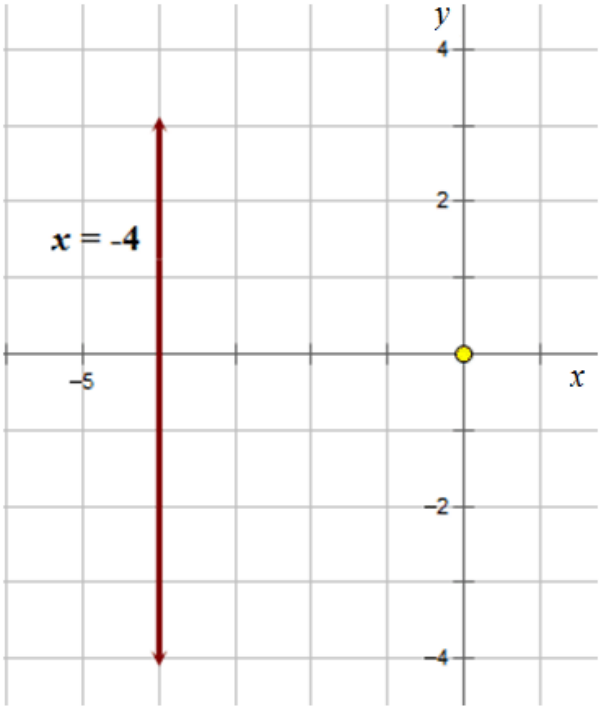
$$\begin{aligned} m &= \frac{-1}{-5} \\ m &= \frac{1}{5} \end{aligned}$$

The slope of the straight line is $\frac{1}{5}$.

Let's look at the slopes of the two special straight lines.

Find the slope of the line having $x = -4$ as its equation.

First, graph the straight line and determine the coordinates of two points that the line passes through.



[Figure 3]

The vertical line passes through the point $(-4, 0)$ and is parallel to the y-axis. The line also passes through the point $(-4, -1)$.

First, name the points as being the first point and the second point.

$$\begin{pmatrix} x_1, & y_1 \\ -4, & 0 \end{pmatrix} \text{ and } \begin{pmatrix} x_2, & y_2 \\ -4, & -1 \end{pmatrix}$$

Next, write the formula for finding the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Next, fill in the values for the coordinates as they have been named.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{-1 - 0}{-4 - -4} \\ m &= \frac{-1 - 0}{-4 + 4} \end{aligned}$$

Next, simplify the numerator and the denominator of the formula.

$$\begin{aligned} m &= \frac{-1}{0} \\ m &= \text{undefined} \end{aligned}$$

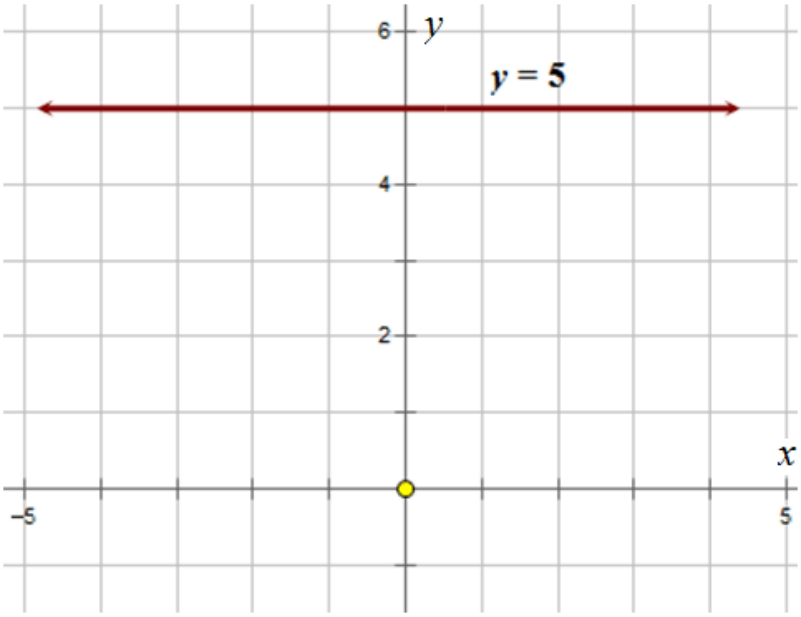
Remember division by zero is undefined.

The slope of the vertical line having the equation $x = -4$ is undefined.

All vertical lines have a slope that is undefined.

Find the slope of the line having $y = 5$ as its equation.

First, graph the straight line and determine the coordinates of two points that the line passes through.



[Figure 4]

The horizontal line passes through the point $(0, 5)$ and is parallel to the x-axis. The line also passes through the point $(1, 5)$.

First, name the points as being the first point and the second point.

$$\begin{pmatrix} x_1, & y_1 \\ 0, & 5 \end{pmatrix} \text{ and } \begin{pmatrix} x_2, & y_2 \\ 1, & 5 \end{pmatrix}$$

Next, write the formula for finding the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Next, fill in the values for the coordinates as they have been named.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{5-5}{1-0} \end{aligned}$$

Next, simplify the numerator and the denominator of the formula.

$$\begin{aligned} m &= \frac{0}{1} \\ m &= 0 \end{aligned}$$

Remember any number divided by zero equals zero.

The slope of the horizontal line having the equation $y = 5$ is zero.

All horizontal lines have a slope of zero.

Examples

Example 1

Earlier, you were given a problem about Jacques and the coordinates on the flags. He needs to figure out if the coordinates have anything to do with the steepness of the ski hill. How can Jacques figure this out?

He can use the formula to calculate the slope of the ski hill.

First, name the points as being the first point and the second point.

$$\begin{pmatrix} x_1, & y_1 \\ 98, & 60 \end{pmatrix} \text{ and } \begin{pmatrix} x_2, & y_2 \\ 82, & 124 \end{pmatrix}$$

Next, write the formula for finding the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Next, fill in the values for the coordinates as they have been named.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{124 - 60}{82 - 98} \end{aligned}$$

Next, simplify the numerator and the denominator of the formula.

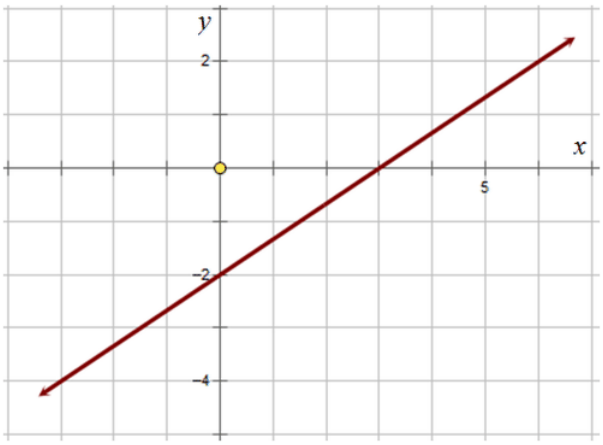
$$\begin{aligned} m &= \frac{64}{-16} \\ m &= \frac{-4}{1} \end{aligned}$$

The slope of the line is $\frac{-4}{1}$.

Remember, the slope of a line indicates the rate at which a change in x creates a change in the value of y. For the ski hill, a horizontal change of positive 1 changes the vertical value 4 units downward. The ski hill is very steep.

Example 2

What is the slope of the straight line shown on the following graph?



[Figure 5]

First, select two exact points on the line.

Next, begin with a point to the left and count horizontally to the right until you are either directly above or directly below the second point.

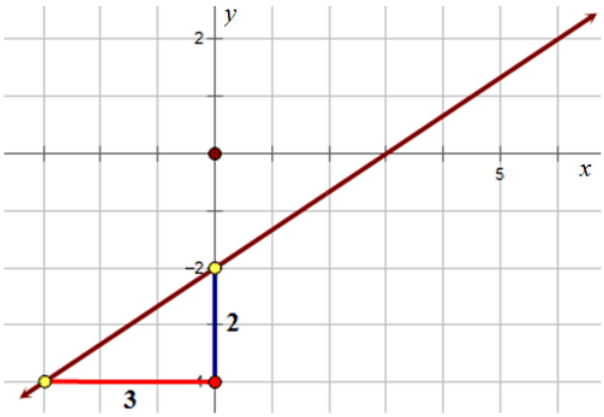
Then, write the number that you have just moved horizontally in a positive direction as the denominator (run) of the slope.

Next, count the number vertically to the second point. If you move vertically upward then the number is positive. If you move vertically downward then the number is negative.

Then, write this value as the numerator (rise) of the slope.

For the above graph:

$$m = \frac{\text{rise}}{\text{run}} \quad m = \frac{2}{3}$$



[Figure 6]

Example 3

What is the slope of the line passing through the points $A(-6, 10)$ and $B(-2, 7)$?

First, name the points as being the first point and the second point.

$$\begin{pmatrix} x_1, & y_1 \\ -6, & 10 \end{pmatrix} \text{ and } \begin{pmatrix} x_2, & y_2 \\ -2, & 7 \end{pmatrix}$$

Next, write the formula for finding the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Next, fill in the values for the coordinates as they have been named.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{7 - 10}{-2 - -6} \\ m &= \frac{7 - 10}{-2 + 6} \end{aligned}$$

Next, simplify the numerator and the denominator of the formula.

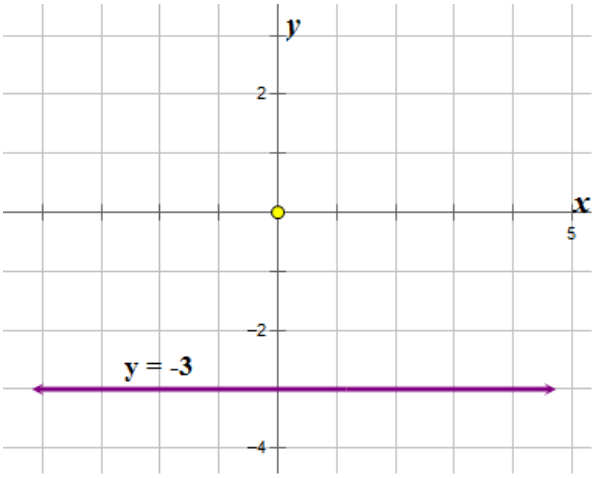
$$m = \frac{-3}{4}$$

The slope of the line is $\frac{-3}{4}$.

Example 4

Describe the graph of a line that has $y = -3$ as its equation and state the slope of the line.

First, draw a graph to model the equation $y = -3$.



[Figure 7]

The graph is a horizontal line parallel to the x-axis. The slope of a horizontal line is zero.

$$m = 0$$

Example 5

Use the formula for slope to calculate the slope of the straight line passing through the points $(-8, -5)$ and $(12, -6)$.

First, name the points as being the first point and the second point.

$$\begin{pmatrix} x_1, & y_1 \\ -8, & -5 \end{pmatrix} \text{ and } \begin{pmatrix} x_2, & y_2 \\ 12, & -6 \end{pmatrix}$$

Next, write the formula for finding the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Next, fill in the values for the coordinates as they have been named.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{-6 - -5}{12 - -8} \\ m &= \frac{-6 + 5}{12 + 8} \end{aligned}$$

Next, simplify the numerator and the denominator of the formula.

$$m = \frac{-1}{20}$$

The slope of the line is $\frac{-1}{20}$.

Review

Answer true or false for each of the following questions.

1. True or false. The equation of a line is always linear.

2. True or false. A linear equation will be shown as a straight line on a graph.

3. True or false. The x-intercept is where the line crosses the y axis.

4. True or false. The y-intercept is where the line crosses the y axis.

5. True or false. A vertical line has an undefined slope.

6. True or false. A horizontal line has a slope of 0.

7. True or false. Slope is the distance that the line travels on the coordinate graph.

8. True or false. Slope is found using a ratio.

9. True or false. You can figure out the slope of a line if you have been given one set of points.

10. True or false. You will need two sets of points that a line passes through to figure out the slope.

Figure out the slope of a line that passes through each of the following pairs of points.

11. $(2, 3)(3, 4)$

12. $(4, 5)(2, 3)$

13. $(2, 1)(-1, 3)$

14. $(3, 1)(4, 3)$

15. $(5, 7)(3, 6)$

16. $(3, 0)(4, 1)$

17. $(6, 4)(2, 7)$

18. $(2, 0)(0, 1)$

19. $(6, 1)(1, 6)$

20. $(4, 4)(5, 0)$

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

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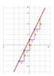	**Credit:** Patrick;Jim Kravitz **Source:** https://www.flickr.com/photos/pdbreen/3373951236/in/photolist-699oa3-pUTbZw-n5Nc9y-qjNUvb-9EyKxF-bBZzd1-7PqDT6-2nruW-25tQi-7PsAR7-b6SqbK-25tPs-6fs9H5-rhcYJo-4cJy6e-96WWnr-qBrfQd-6fs9m7-6fs8X3-s9AxC6-brHiYG-5Z4uXE-7By631-biDxjk-cf7U-7HsrSY-BP2bM-dNYGYR-4mS4V8-bmuLKk-qut3UH-bqyngv-9D1mpz-7oMahh-7oM4Yy-6w5jq-qGJmKN-a3bCJh-EH3iY-8Smni-n5KZcx-eyUmv-yaZCn-4ug5r8-kcxPn-7TCnPu-GKhEX-h4bKh-4DNmyG-7xfT8p ; https://www.flickr.com/photos/jimmypk/4522379334/in/photolist-7TCnPu-GKhEX-h4bKh-4DNmyG-7xfT8p-7xfSYK-b1SqFR-9sCLxr-6w4MU-aZaig4-jzMKB1-4x1j81-kcxNV-4Wh1th-64nnMR-25tRs-jRHT6-EH2BF-7xfQYH-6w5h2-4zX5Qv-96WWdc-reGHNZ-jzNRfL-5ZLy9Q-8dF4q-wiZ4r-4u7aAQ-n5LdKp-4DNptC-4DJ3XX-9CRUaZ-9sYcux-aQKTz-ytV4J-64rxjs-uj77D-9j462S-25tQD-bxUiem-kcE4Uz-4DJ6AH-6w5zQ-5ybJyp-bACa1m-9orGUo-9ooDAF-7JoYVU-beAsi8-9CUQjG
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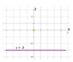	**Credit:** Patrick;Jim Kravitz **Source:** https://www.flickr.com/photos/pdbreen/3373951236/in/photolist-699oa3-pUTbZw-n5Nc9y-qjNUvb-9EyKxF-bBZzd1-7PqDT6-2nruW-25tQi-7PsAR7-b6SqbK-25tPs-6fs9H5-rhcYJo-4cJy6e-96WWnr-qBrfQd-6fs9m7-6fs8X3-s9AxC6-brHiYG-5Z4uXE-7By631-biDxjk-cf7U-7HsrSY-BP2bM-dNYGYR-4mS4V8-bmuLKk-qut3UH-bqyngv-9D1mpz-7oMahh-7oM4Yy-6w5jq-qGJmKN-a3bCJh-EH3iY-8Smni-n5KZcx-eyUmv-yaZCn-4ug5r8-kcxPn-7TCnPu-GKhEX-h4bKh-4DNmyG-7xfT8p ; https://www.flickr.com/photos/jimmypk/4522379334/in/photolist-7TCnPu-GKhEX-h4bKh-4DNmyG-7xfT8p-7xfSYK-b1SqFR-9sCLxr-6w4MU-aZaig4-jzMKB1-4x1j81-kcxNV-4Wh1th-64nnMR-25tRs-jRHT6-EH2BF-7xfQYH-6w5h2-4zX5Qv-96WWdc-reGHNZ-jzNRfL-5ZLy9Q-8dF4q-wiZ4r-4u7aAQ-n5LdKp-4DNptC-4DJ3XX-9CRUaZ-9sYcux-aQKTz-ytV4J-64rxjs-uj77D-9j462S-25tQD-bxUiem-kcE4Uz-4DJ6AH-6w5zQ-5ybJyp-bACa1m-9orGUo-9ooDAF-7JoYVU-beAsi8-9CUQjG