Use the Pythagorean Theorem

Brenda Meery
Jen Kershaw

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AUTHORS
Brenda Meery
Jen Kershaw

8.4 Use the Pythagorean Theorem

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[Figure 1]

The owners of a campground have just purchased a small island that they hope to add to their property as a spot for campers who use tents. First, the owners need to build a bridge to connect the two properties. The river is 34 feet wide and the higher land on the island is 13 feet from the river. How can the owners figure out the length of the bridge they need to build?

In this concept, you will learn to use the Pythagorean Theorem.

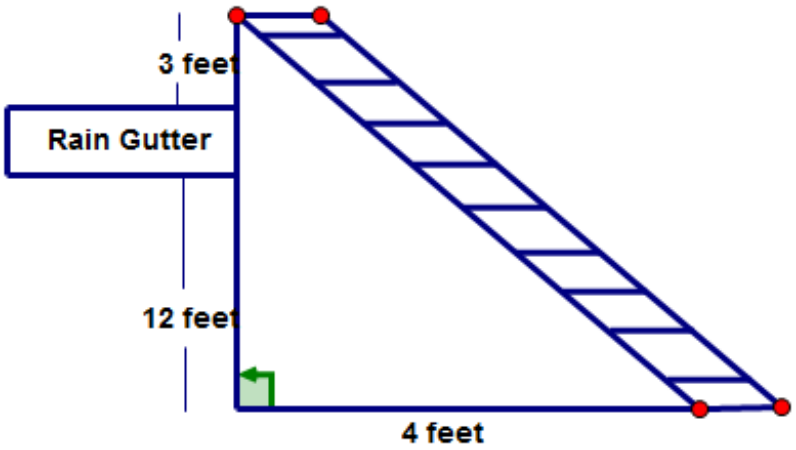
Indirect Measurement

In the real world, it is often not possible to use determine distances or lengths using rulers, tape measures or other measuring devices. When direct measurement is not an option, indirect measurement is used. **Indirect measurement** is an approach to measuring using alternative measurements and properties that will assist in determining the unknown distances or lengths needed to be found. The properties of the Pythagorean Theorem are often used when something needs to be indirectly measured.

Consider the following situation.

Jim must clean the rain gutters of his house. The gutters are 12 feet above the ground. He knows the ladder must extend 3 feet above the gutters so he can hold unto the rails as he cleans. If he places the foot of the ladder 4 feet from the base of the house, what length should the ladder be that he uses?

First, draw a diagram to represent the information.



[Figure 2]

Next, determine the values for (a, b, c) of the Pythagorean Theorem.

$$\begin{aligned} a &= 12 \text{ feet} + 3 \text{ feet} = 15 \\ b &= 4 \text{ feet} = 4 \\ c &= \text{length of ladder} \end{aligned}$$

Next, fill the values into the Pythagorean Theorem.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= (15)^2 + (4)^2 \end{aligned}$$

Next, perform the indicated squaring and simplify the equation.

$$\begin{aligned} c^2 &= (15)^2 + (4)^2 \\ c^2 &= (15 \times 15) + (4 \times 4) \\ c^2 &= 225 + 16 \\ c^2 &= 241 \end{aligned}$$

Then, solve for c by taking the square root of both sides of the equation.

$$\begin{aligned} c^2 &= 241 \\ \sqrt{c^2} &= \sqrt{241} \\ c &= 15.52 \end{aligned}$$

The answer is 15.52.

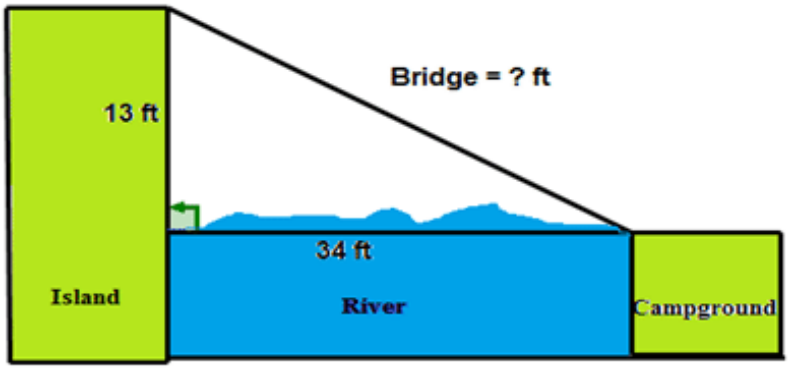
Jim needs to use a 16 foot ladder.

Examples

Example 1

Earlier, you were given a problem about the campgrounds and the island. The owners have to determine the length of the bridge they need to build to connect the campgrounds to the island. How can they calculate this indirect measurement using the Pythagorean Theorem?

First, draw and label a right triangle to model the problem.



[Figure 3]

Next, determine the values of (a, b, c) for the Pythagorean Theorem.

$$\begin{aligned} a &= 13\ ft &&= 13 \\ b &= 34\ ft &&= 34 \\ c &= ?\ ft &&= c \end{aligned}$$

Next, fill the values into the Pythagorean Theorem.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= (13)^2 + (34)^2 \end{aligned}$$

Next, perform the indicated squaring and simplify the equation.

$$\begin{aligned} c^2 &= 13^2 + (34)^2 \\ c^2 &= (13 \times 13) + (34 \times 34) \\ c^2 &= 169 + 1156 \\ c^2 &= 1325 \end{aligned}$$

Then, solve for c by taking the square root of both sides of the equation.

$$\begin{aligned} c^2 &= 1325 \\ \sqrt{c^2} &= \sqrt{1325} \\ c &= 36.4 \end{aligned}$$

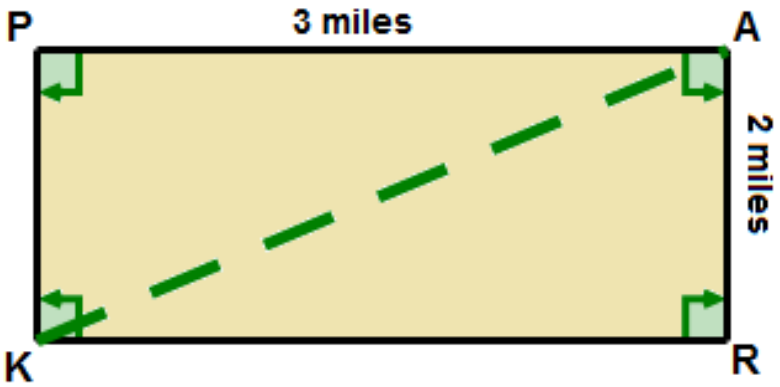
The answer is 36.4.

The length of the bridge must be 36.4 feet.

Example 2

The new Sunset Walking Park has just opened and you would like to take your mom to see it. The park is in the shape of a rectangle measuring 2 miles by 3 miles. You don't know whether to take your mom for a walk around the outside perimeter of the park or to take her along the diagonal pathway. Which way would be shorter and by how much?

First, draw and label a diagram to represent the park.



[Figure 4]

You can see that the diagonal pathway divides the rectangle into two right triangles. Use Pythagorean Theorem to find the length of the diagonal pathway.

Next, determine the values of (a, b, c) for the Pythagorean Theorem.

$$\begin{aligned} a &= 3 \text{ miles} = 3 \\ b &= 2 \text{ miles} = 2 \\ c &= ? \text{ miles} = c \end{aligned}$$

Next, fill the values into the Pythagorean Theorem.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= (3)^2 + (2)^2 \end{aligned}$$

Next, perform the indicated squaring and simplify the equation.

$$\begin{aligned} c^2 &= (3)^2 + (2)^2 \\ c^2 &= (3 \times 3) + (2 \times 2) \\ c^2 &= 9 + 4 \\ c^2 &= 13 \end{aligned}$$

Then, solve for c by taking the square root of both sides of the equation.

$$\begin{aligned} c^2 &= 13 \\ \sqrt{c^2} &= \sqrt{13} \\ c &= 3.6 \end{aligned}$$

The answer is 3.6.

The length of the pathway is 3.6 miles.

The distance around the park is the sum of the lengths of the four sides of the rectangle.

$$\begin{aligned} d &= 3 + 2 + 3 + 2 \\ d &= 10 \end{aligned}$$

The answer is 10.

The distance around the park is 10 miles.

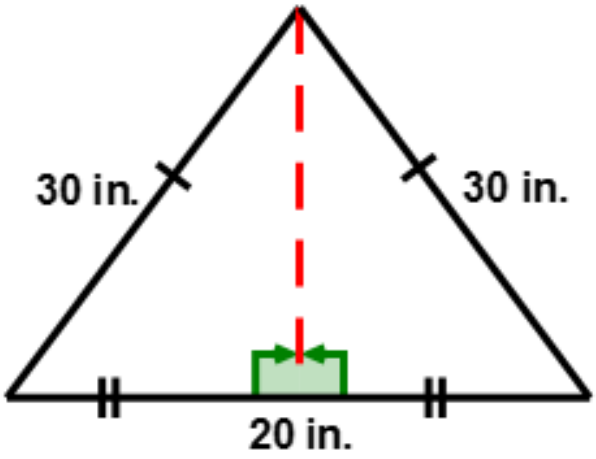
$$10.0 \text{ miles} - 3.6 \text{ miles} = 6.4 \text{ miles}$$

The answer is 6.4 miles.

Example 3

An isosceles triangle has sides 30 inches, 30 inches and 20 inches. Find the length altitude to the shortest side of the triangle using Pythagorean Theorem. Express the length of the altitude to the nearest inch.

First, draw and label a diagram to represent the problem.



[Figure 5]

The altitude of an isosceles triangle cuts the base of the triangle into two equal lengths and creates two right triangles.

Next, determine the values of (a, b, c) for the Pythagorean Theorem.

$$\begin{aligned} a &= 10 \text{ in} = 10 \\ b &= ? \text{ in} = b \\ c &= 30 \text{ in} = 30 \end{aligned}$$

Next, fill the values into the Pythagorean Theorem.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ (30)^2 &= (10)^2 + b^2 \end{aligned}$$

Next, perform the indicated squaring and simplify the equation.

$$\begin{aligned} (30)^2 &= (10)^2 + b^2 \\ (30 \times 30) &= (10 \times 10) + b^2 \\ 900 &= 100 + b^2 \end{aligned}$$

Next, isolate the variable by subtracting 100 from both sides of the equation and simplify the equation.

$$\begin{aligned} 900 &= 100 + b^2 \\ 900 - 100 &= 100 - 100 + b^2 \\ 800 &= b^2 \end{aligned}$$

Then, solve for the variable by taking the square root of both sides of the equation.

$$\begin{aligned} 800 &= b^2 \\ \sqrt{800} &= \sqrt{b^2} \\ 28.28 &= b \end{aligned}$$

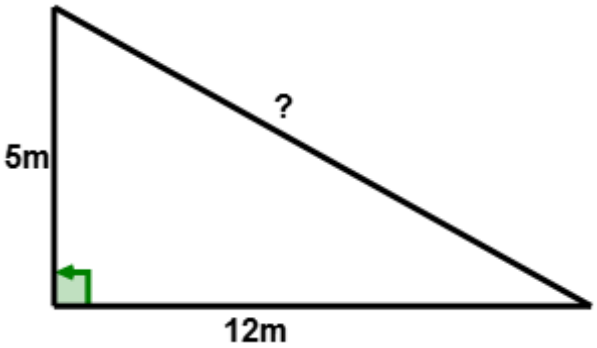
The answer is 28.

The length of the altitude is 28 inches.

Example 4

Find the length of the hypotenuse of a right triangle when the lengths of the other two sides are 5 m and 12 m.

First, draw and label a right triangle to model the problem.



[Figure 6]

Next, determine the values of (a, b, c) for the Pythagorean Theorem.

$$\begin{aligned} a &= 5\ m = 5 \\ b &= 12\ m = 12 \\ c &= ?\ m = c \end{aligned}$$

Next, fill the values into the Pythagorean Theorem.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= (5)^2 + (12)^2 \end{aligned}$$

Next, perform the indicated squaring and simplify the equation.

$$\begin{aligned} c^2 &= (5)^2 + (12)^2 \\ c^2 &= (5 \times 5) + (12 \times 12) \\ c^2 &= 25 + 144 \\ c^2 &= 169 \end{aligned}$$

Then, solve for c by taking the square root of both sides of the equation.

$$\begin{aligned} c^2 &= 169 \\ \sqrt{c^2} &= \sqrt{169} \\ c &= 13 \end{aligned}$$

The answer is 13.

The length of the hypotenuse is 13 m.

Review

Use the Pythagorean Theorem to determine if each set of measurements could represent a right triangle. Sets of integers that could represent the sides of right triangles are called Pythagorean Triples.

1. 3, 4, 5

2. 6, 8, 12

3. 6, 8, 10

4. 15, 20, 25

5. 5, 9, 14

6. 9, 12, 15

7. 18, 24, 30

8. 5, 12, 13

9. 7, 24, 25

10. 8, 15, 17

Find the missing side length of each right triangle by using the Pythagorean Theorem. You may round to the nearest tenth when necessary.

11. $a = 6, b = 10, c =$ _____

12. $a = 5, b = 7, c =$ _____

13. $a = 7, b = 9, c =$ _____

14. $a = 6, b = 8, c =$ _____

15. $a = 9, b = 12, c =$ _____

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

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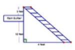
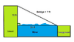
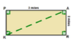
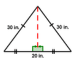
	**Credit:** oakleyOriginals;Greg Hayter **Source:** https://www.flickr.com/photos/oakleyoriginals/4854002700/in/photolist-8oW2TJ-fM3st6-pPaD9H-9SnTqx-9SqQ6f-9SqPHo-9SqMbh-38o1wy-9SnVtP-nvpSYs-9SnVPB-4SQUB5-9EMFWt-57VqaS-no2vL-6LP41c-5xZh8g-6Bp1sd-ejCenN-9SqQTW-2R6Pda-a2s32b-an3PPG-7KMDHm-dku91G-5828co-6CuDe3-et2irZ-9SnUue-9SnYdx-96MFzo-6rwRKY-g8pwCP-6M1q7x-dtjsU9-4FWpxB-58GXwY-2rAykD-aDy4Hq-bRaXqV-oD5WWf-oD5e2p-8mdXRa-5e63Wt-davSai-72bzHF-4uBRFP-cpxUmh-9q8rVv-5dLSTH ; https://www.flickr.com/photos/gphayter/3647497352/in/photolist-pNJQzo-ffxruV-d8Rbk3-9tZR3T-cYAskm-9WwTzp-cFt3od-dtf2Vm-rrQJxo-dmDtFU-mh5Nw8-r7kb7b-oBQ7zj-6yjnTo-ghWsGP-as8Ff6-qxgR5j-e5JnM2-7QP5Ph-dYu2aR-d8PZH7-ghWsTF-spxna8-84r6ip-pBf9TR-anTND6-i6tnjY-d8Pq7J-bmRtAv-ao6JX6-rNPE6h-d8Q66m-h7LFj3-ahgfoP-9in4TS-h7LFZS-dSP4tp-aizopX-cGcWsh-q3obS9-dxC5wk-5z2ngW-5z2mbW-7GMBgH-9PLcxL-gArQuV-j1jxnU-gs8BLe-nTRWsB-ps9bXJ
	**Credit:** oakleyOriginals;Greg Hayter **Source:** https://www.flickr.com/photos/oakleyoriginals/4854002700/in/photolist-8oW2TJ-fM3st6-pPaD9H-9SnTqx-9SqQ6f-9SqPHo-9SqMbh-38o1wy-9SnVtP-nvpSYs-9SnVPB-4SQUB5-9EMFWt-57VqaS-no2vL-6LP41c-5xZh8g-6Bp1sd-ejCenN-9SqQTW-2R6Pda-a2s32b-an3PPG-7KMDHm-dku91G-5828co-6CuDe3-et2irZ-9SnUue-9SnYdx-96MFzo-6rwRKY-g8pwCP-6M1q7x-dtjsU9-4FWpxB-58GXwY-2rAykD-aDy4Hq-bRaXqV-oD5WWf-oD5e2p-8mdXRa-5e63Wt-davSai-72bzHF-4uBRFP-cpxUmh-9q8rVv-5dLSTH ; https://www.flickr.com/photos/gphayter/3647497352/in/photolist-pNJQzo-ffxruV-d8Rbk3-9tZR3T-cYAskm-9WwTzp-cFt3od-dtf2Vm-rrQJxo-dmDtFU-mh5Nw8-r7kb7b-oBQ7zj-6yjnTo-ghWsGP-as8Ff6-qxgR5j-e5JnM2-7QP5Ph-dYu2aR-d8PZH7-ghWsTF-spxna8-84r6ip-pBf9TR-anTND6-i6tnjY-d8Pq7J-bmRtAv-ao6JX6-rNPE6h-d8Q66m-h7LFj3-ahgfoP-9in4TS-h7LFZS-dSP4tp-aizopX-cGcWsh-q3obS9-dxC5wk-5z2ngW-5z2mbW-7GMBgH-9PLcxL-gArQuV-j1jxnU-gs8BLe-nTRWsB-ps9bXJ
	**Credit:** oakleyOriginals;Greg Hayter **Source:** https://www.flickr.com/photos/oakleyoriginals/4854002700/in/photolist-8oW2TJ-fM3st6-pPaD9H-9SnTqx-9SqQ6f-9SqPHo-9SqMbh-38o1wy-9SnVtP-nvpSYs-9SnVPB-4SQUB5-9EMFWt-57VqaS-no2vL-6LP41c-5xZh8g-6Bp1sd-ejCenN-9SqQTW-2R6Pda-a2s32b-an3PPG-7KMDHm-dku91G-5828co-6CuDe3-et2irZ-9SnUue-9SnYdx-96MFzo-6rwRKY-g8pwCP-6M1q7x-dtjsU9-4FWpxB-58GXwY-2rAykD-aDy4Hq-bRaXqV-oD5WWf-oD5e2p-8mdXRa-5e63Wt-davSai-72bzHF-4uBRFP-cpxUmh-9q8rVv-5dLSTH ; https://www.flickr.com/photos/gphayter/3647497352/in/photolist-pNJQzo-ffxruV-d8Rbk3-9tZR3T-cYAskm-9WwTzp-cFt3od-dtf2Vm-rrQJxo-dmDtFU-mh5Nw8-r7kb7b-oBQ7zj-6yjnTo-ghWsGP-as8Ff6-qxgR5j-e5JnM2-7QP5Ph-dYu2aR-d8PZH7-ghWsTF-spxna8-84r6ip-pBf9TR-anTND6-i6tnjY-d8Pq7J-bmRtAv-ao6JX6-rNPE6h-d8Q66m-h7LFj3-ahgfoP-9in4TS-h7LFZS-dSP4tp-aizopX-cGcWsh-q3obS9-dxC5wk-5z2ngW-5z2mbW-7GMBgH-9PLcxL-gArQuV-j1jxnU-gs8BLe-nTRWsB-ps9bXJ
	**Credit:** oakleyOriginals;Greg Hayter **Source:** https://www.flickr.com/photos/oakleyoriginals/4854002700/in/photolist-8oW2TJ-fM3st6-pPaD9H-9SnTqx-9SqQ6f-9SqPHo-9SqMbh-38o1wy-9SnVtP-nvpSYs-9SnVPB-4SQUB5-9EMFWt-57VqaS-no2vL-6LP41c-5xZh8g-6Bp1sd-ejCenN-9SqQTW-2R6Pda-a2s32b-an3PPG-7KMDHm-dku91G-5828co-6CuDe3-et2irZ-9SnUue-9SnYdx-96MFzo-6rwRKY-g8pwCP-6M1q7x-dtjsU9-4FWpxB-58GXwY-2rAykD-aDy4Hq-bRaXqV-oD5WWf-oD5e2p-8mdXRa-5e63Wt-davSai-72bzHF-4uBRFP-cpxUmh-9q8rVv-5dLSTH ; https://www.flickr.com/photos/gphayter/3647497352/in/photolist-pNJQzo-ffxruV-d8Rbk3-9tZR3T-cYAskm-9WwTzp-cFt3od-dtf2Vm-rrQJxo-dmDtFU-mh5Nw8-r7kb7b-oBQ7zj-6yjnTo-ghWsGP-as8Ff6-qxgR5j-e5JnM2-7QP5Ph-dYu2aR-d8PZH7-ghWsTF-spxna8-84r6ip-pBf9TR-anTND6-i6tnjY-d8Pq7J-bmRtAv-ao6JX6-rNPE6h-d8Q66m-h7LFj3-ahgfoP-9in4TS-h7LFZS-dSP4tp-aizopX-cGcWsh-q3obS9-dxC5wk-5z2ngW-5z2mbW-7GMBgH-9PLcxL-gArQuV-j1jxnU-gs8BLe-nTRWsB-ps9bXJ
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