Multiplying Binomials

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9.6 Multiplying Binomials

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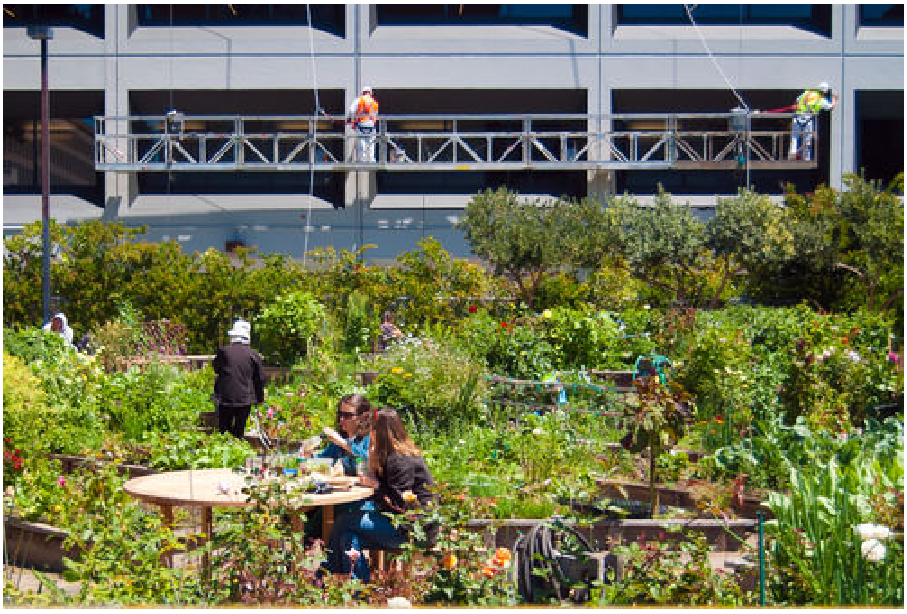
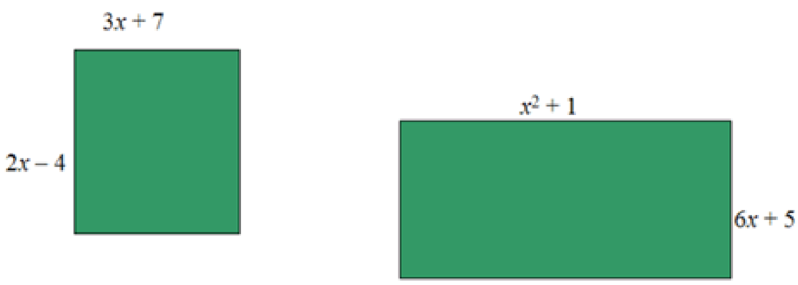


[Figure 1]

Brenda and her mother are walking through their local community garden, enjoying the gentle aromas and beautiful colors on display. While her mother talks to one of the gardeners, Brenda looks through the brochure with all the garden dimensions and comes up with a math challenge for her mother:

If one field dedicated to vegetables measures $3x+7$ by $2x-4$, while another field dedicated to flowers measures x^2+1 by $6x+5$, what is the combined area of the two fields?



[Figure 2]

In this concept, you will learn to multiply binomials vertically, horizontally and by using a table.

Multiplying Binomials

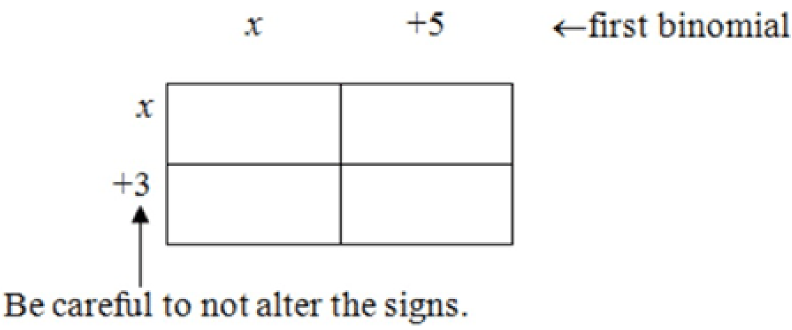
Binomials are defined as two-term polynomials. When you add and subtract polynomials, you are careful to combine like terms. When you multiply polynomials you will carefully apply the rules of exponents, as well.

When you multiply binomials, you can use a table to help us to organize and keep track of the information.

Let's look at an example.

Multiply the binomials $(x+5)(x+3)$.

First, use a table like a rectangle, as if each of the binomials were a dimension of the rectangle. You will insert the two binomials along the sides of the table like a rectangle.



[Figure 3]

Next, find the area of the four separate rectangles.

	x	$+5$
x	x^2	$5x$
$+3$	$3x$	15

[Figure 4]

The dimensions of the first rectangle are $x \times x$, while dimensions of the second are $5 \times x$, the third are $3 \times x$, and the fourth are 3×5 .

Then, in order to find the total, you will add the four areas and combine like terms.

$$x^2 + 5x + 3x + 15$$
$$x^2 + 8x + 15$$

The answer is $x^2 + 8x + 15$.

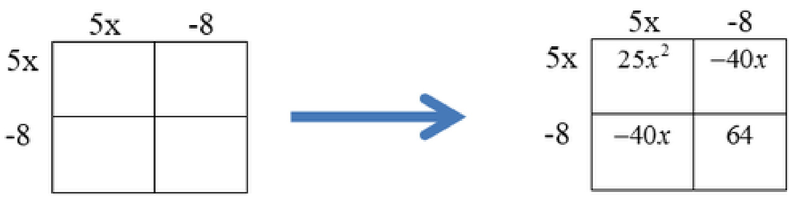
Here is an example that is a little different.

Multiply $(5x-8)^2$.

First, remember that the exponent applies to the entire binomial such that

$$(5x-8)^2 = (5x-8)(5x-8)$$

Fill in the table below with the areas of each of the rectangles.



[Figure 5]

Next, in order to find the total, you will add the four areas and combine like terms.

$$\begin{aligned}&25x^2-40x-40x+64\\&25x^2-80x+64\end{aligned}$$

The answer is $25x^2-80x+64$.

A second method for multiplying binomials is similar to the algorithm commonly used for multiplying two- digit numbers. You can multiply binomials vertically in the same manner.

Let's take a look at an example.

Multiply:

$$(3x+2)(5x+4)$$

First, set up the vertical multiplication.

$$\begin{array}{lccc} 2^{nd}\text{ power} & 1^{st}\text{ power} & & 0\text{ power}\\ & 3x & + & 2\\ \times & 5x & + & 4\\ \hline \end{array}$$

Next, complete the vertical multiplication.

2^{nd} power		1^{st} power		0 power
		$3x$	$+$	2
		$5x$	$+$	4
		$12x$	$+$	8
$15x^2$	$+$	$10x$		
$15x^2$	$+$	$22x$	$+$	8

The answer is $15x^2+22x+8$.

A third way is to use "**FOIL**". **FOIL** is an acronym which tells you which terms to multiply in order to get the product of the two binomials.

FOIL stands for:

F: First terms in the binomials

O: Outside terms in the binomials

I: Inside terms in the binomials

L: Last terms in the binomials

Let's look at an example.

Multiply using the FOIL method.

$$(2x+8)(5x-13)$$

First, multiply the first two terms.

$$({\color{red}2x}+8)({\color{red}5x}-13)$$

First terms: $2x \times 5x = 10x^2$

Next, multiply the outside terms.

$$({\color{red}2x}+8)(5x{\color{red}-13})$$

Outside terms: $2x \times -13 = -26x$

Next, multiply the inside terms.

$$(2x + 8)(5x - 13)$$

Inside terms: $8 \times 5x = 40x$

Then, multiply the last two terms.

$$(2x + 8)(5x - 13)$$

Last terms: $8 \times -13 = -104$

Then, combine like terms.

$$\begin{aligned}(2x + 8)(5x - 13) &= 10x^2 - 26x + 40x - 104 \\ &= 10x^2 + 14x - 104\end{aligned}$$

The answer is $10x^2 + 14x - 104$.

Of the three methods in this concept for multiplication, you might agree that this is the quickest method. Of course, all three methods would give you the same product.

Take a look at one more.

Multiply using the FOIL method.

$$(5x^3 + 2x)(7x^2 + 8)$$

First, use FOIL to find the four terms that result from multiplying the two binomials.

$$\begin{aligned}(5x^3 + 2x)(7x^2 + 8) &= 5x^3 \times 7x^2 + 5x^3 \times 8 + 2x \times 7x^2 + 2x \times 8 \\ &= 35x^5 + 40x^3 + 14x^3 + 16x\end{aligned}$$

Next, combine like terms.

$$35x^5 + 40x^3 + 14x^3 + 16x = 35x^5 + 54x^3 + 16x$$

The answer is $35x^5 + 54x^3 + 16x$.

Examples

Example 1

Earlier, you were given a problem about Brenda's math challenge for her mother.

Brenda ends up helping her mother work out the area of both rectangles and then the sum of those two areas to get the total area.

First, they find the area of the first rectangle using FOIL.

$$\begin{aligned}(3x+7)(2x-4) &= 3x \times 2x + 3x \times -4 + 7 \times 2x + 7 \times -4 \\ &= 6x^2 - 12x + 14x - 28 \\ &= 6x^2 + 2x - 28\end{aligned}$$

Next, they find the area of the second rectangle using FOIL.

$$\begin{aligned}(x^2+1)(6x+5) &= x^2 \times 6x + x^2 \times 5 + 1 \times 6x + 1 \times 5 \\ &= 6x^3 + 5x^2 + 6x + 5\end{aligned}$$

Then, they add the two areas together to find the total area.

$$\begin{aligned}\text{Total Area} &= (6x^2 + 2x - 28) + (6x^3 + 5x^2 + 6x + 5) \\ &= 6x^3 + 11x^2 + 8x - 23\end{aligned}$$

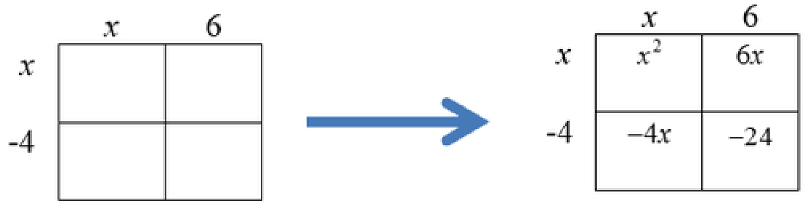
The answer is $6x^3 + 11x^2 + 8x - 23$.

Example 2

Multiply by using a table.

$$(x-4)(x-6)$$

First, fill in the table below with the areas of each of the rectangles.



[Figure 6]

Next, in order to find the total, you will add the four areas and combine like terms.

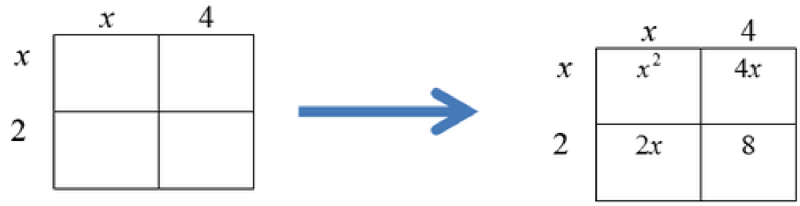
$$x^2+6x-4x-24$$
$$x^2+2x-24$$

The answer is $x^2+2x-24$.

Example 3

Multiply $(x+2)(x+4)$.

First, fill in the table below with the areas of each of the rectangles.



[Figure 7]

Next, in order to find the total, you will add the four areas and combine like terms.

$$x^2+4x+2x+8$$
$$x^2+6x+8$$

The answer is x^2+6x+8.

Example 4

Multiply $(x-6)(x+5)$.

First, use FOIL to find the four terms that result from multiplying the two binomials.

$$\begin{aligned}(x-6)(x+5) &= x \times x + x \times 5 - 6 \times x - 6 \times 5 \\ &= x^2 + 5x - 6x - 30\end{aligned}$$

Next, combine like terms.

$$x^2 + 5x - 6x - 30 = x^2 - x - 30$$

The answer is $x^2 - x - 30$.

Example 5

Multiply $(x-3)(x+3)$.

First, use FOIL to find the four terms that result from multiplying the two binomials.

$$\begin{aligned}(x-3)(x+3) &= x \times x + x \times 3 - 3 \times x - 3 \times 3 \\ &= x^2 + 3x - 3x - 9\end{aligned}$$

Next, combine like terms.

$$x^2 + 3x - 3x - 9 = x^2 - 9$$

The answer is $x^2 - 9$.

Review

Use a table to multiply the following binomials.

1. $(x+3)(x+5)$
2. $(x-3)(x-5)$
3. $(x+3)(x-3)$
4. $(x+2)(x-8)$
5. $(3x^2+3x)(6x-2)$

6. $(2x - 7y)(5x + 4y)$
7. $(2x - 9)^2$

Multiply the following binomials vertically.

8. $(d + 2)(4d - 1)$
9. $(5x + 7)(5x - 7)$
10. $(4b^2 + 3c)(2b - 5c^2)$

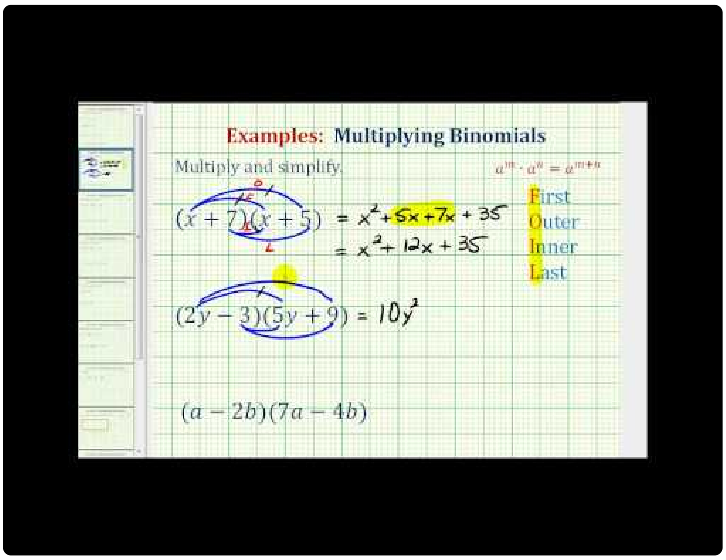
Multiply the following binomials using the FOIL method.

11. $(p + 6)(5p + 2)$
12. $(-7y^2 - 4y)(6y + 2)$
13. $(x^3 + 3x)^2$
14. $(2x + 1)(x - 4)$
15. $(3x - 3)(5x + 9)$
16. $(x + 5)^2$

Review (Answers)

To see the review answers, return to the Table of Contents and select 'Other Versions' or 'Resources'.

Resources



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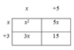	**Credit:** Mosman Council **Source:** https://www.flickr.com/photos/mosmancouncil/8355137887/in/photolist-dJjgpv-5BggiV-5qcd85-ccT7d3-duRvSw-duKPkB-duRzz7-duKW4e-duRnbE-duKYnH-duRxnC-duKPYK-duKUBp-cN2CMf-bQENNM-58V8vw-58V4Qo-bAN9Dw-cmvjQq-eanTi2-bzNgGY-7Wnbc5-duKXBa-duKMfP-duKRR4-duRrCU-duKTT6-duKNMH-duKT4R-duKQCT-duRsQq-duKZTH-duRoDY-gmEUP7-aoC8oM-fGx4pG-dJjhLp-dJjgTD-dJjh4K-dJjhUp-dJpGWm-8xsTn4-5n7E3o-58V5VE-58Veem-bxyYVq-bLtErM-m7WNen-gPdrKY-dq7cix
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